Operations research: a connecting bridge

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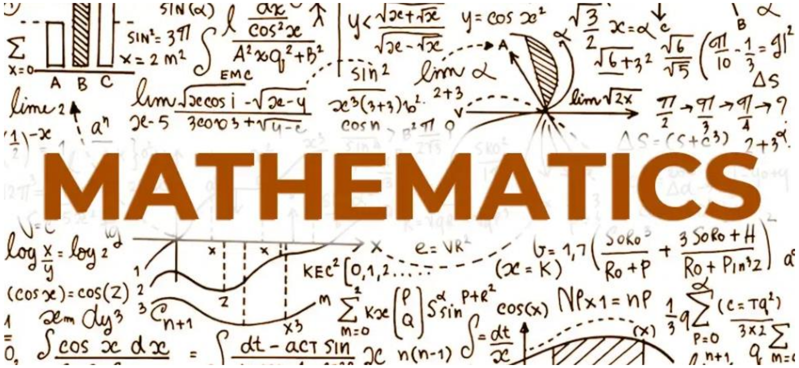
Why Operations Research?



Operations research and technology

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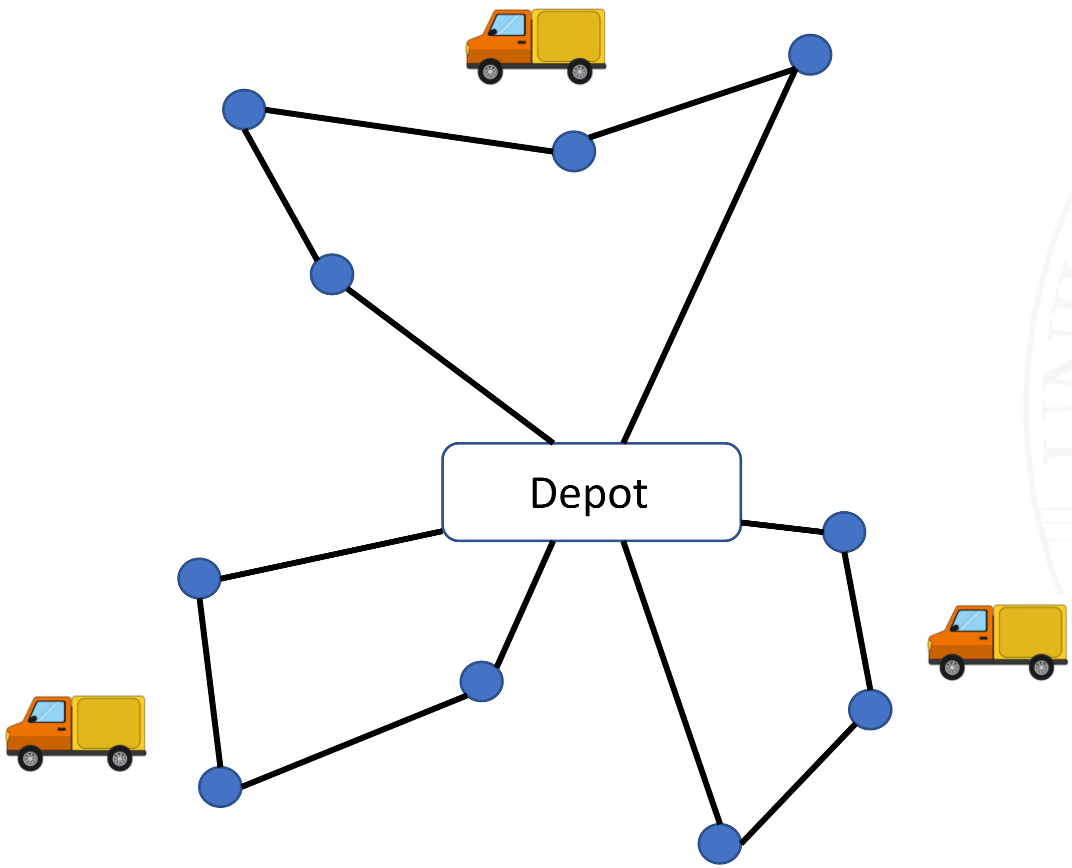
My papers

Connecting decisions



Vehicle routing problems

Connecting decisions

Vehicle routing problems

Decisions

Who?

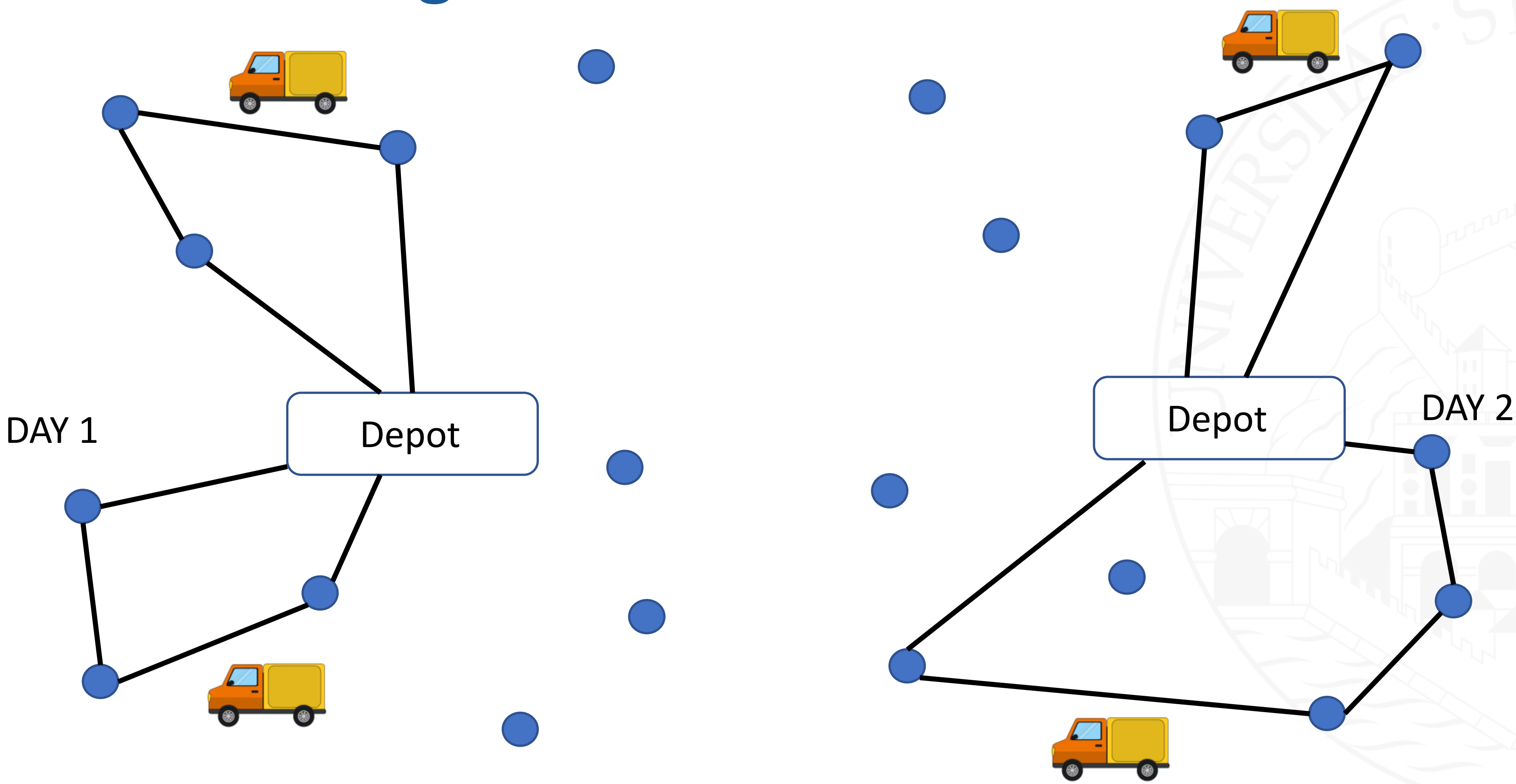
In which order?

+

When?

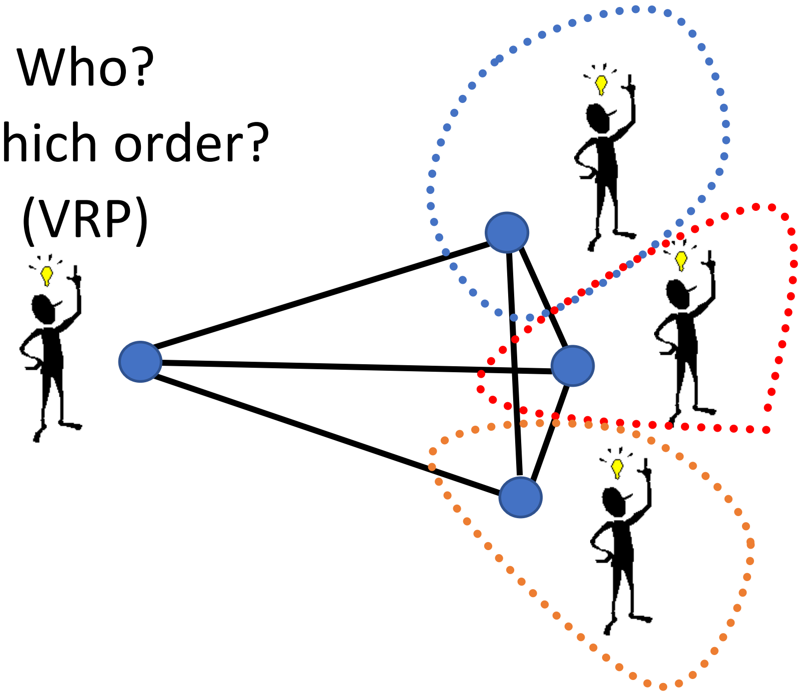
How much?

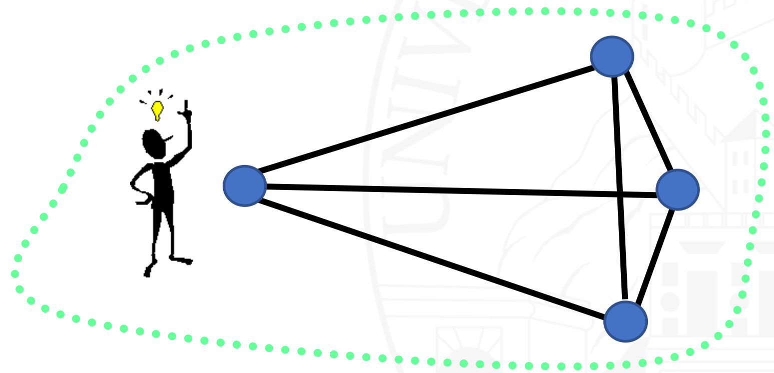
Connecting decisions



Inventory routing problems

Connecting decisions





Vendor managed inventory
and a more complex problem

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Connecting decisions

Instances: up to 50 customers, 6 days
Vehicle routing problems: **optimal** solution
Inventory routing problem: **heuristic** solution

Savings with the same final inventory levels
Average total cost: 10% (max 20%)
Average number of routes: 12% (max 50%)

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Mixed integer programming and finance

Capital C

Assets 1,…,j,…,n

Decision variables x_j

No risk function can be expressed in linear form directly through variables x_j

Mixed integer programming and finance

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j$$

$$\sum_{j=1}^{n} r_j x_j \geq \mu_0$$

$$\sum_{j=1}^{n} x_j = 1$$

$$x_j \geq 0 \quad j = 1,\ldots,n$$

Markowitz's model

Harry Markowitz was the recipient of the 1990 Nobel Prize in Economic Sciences (with Merton Miller and William Sharpe "for their pioneering work in the theory of financial economics“)

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Mixed integer programming and finance

A new modelling approach

T scenarios with probabilities p_t

Discretization

$$\boxed{R_j \to r_{jt}} \qquad r_j = \sum_{t=1}^{T} r_{jt} p_t \qquad y_t = \sum_{j=1}^{n} r_{jt} x_j$$

$$R_{\mathbf{x}} \to \sum_{j=1}^{n} r_{jt} x_j \qquad E[R_{\mathbf{x}}] = \sum_{t=1}^{T} p_t y_t = \sum_{j=1}^{n} r_j x_j$$

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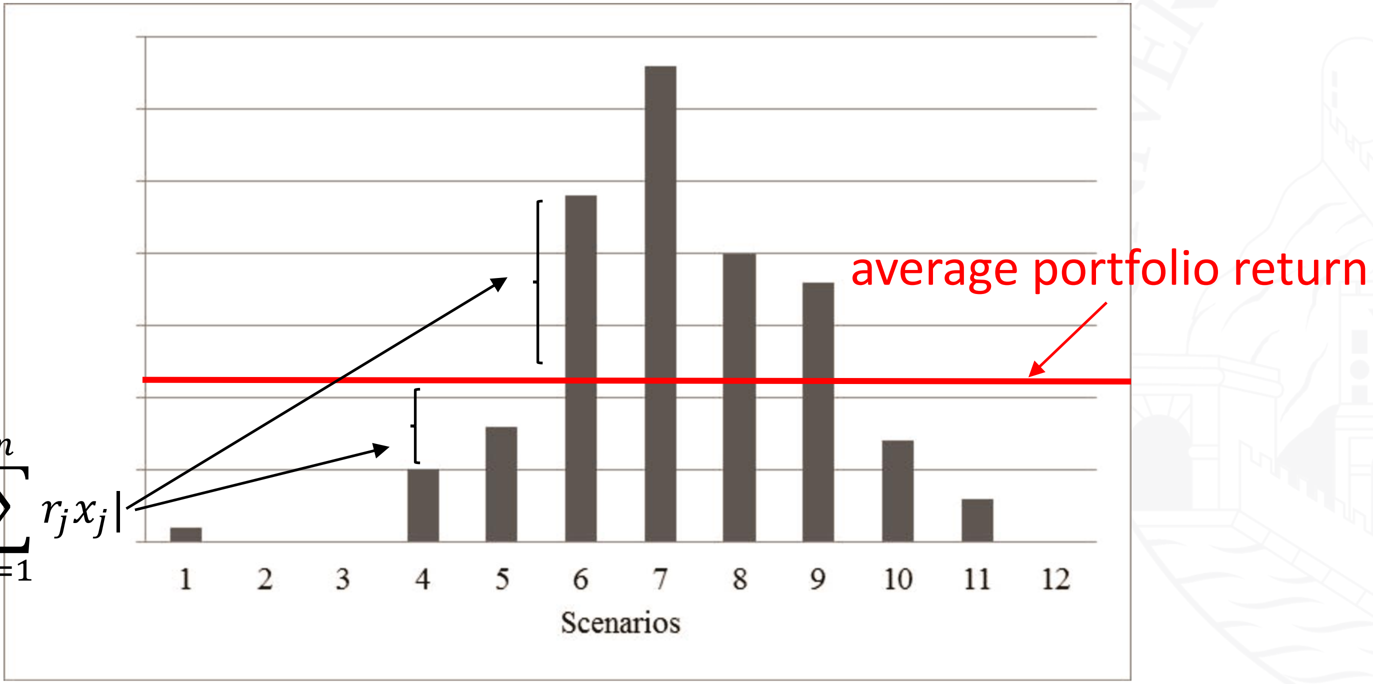
Mixed integer programming and finance

$$d_t = \left|\sum_{j=1}^{n} r_{jt}x_j - \sum_{j=1}^{n} r_j x_j\right|$$



Mean Absolute Deviation

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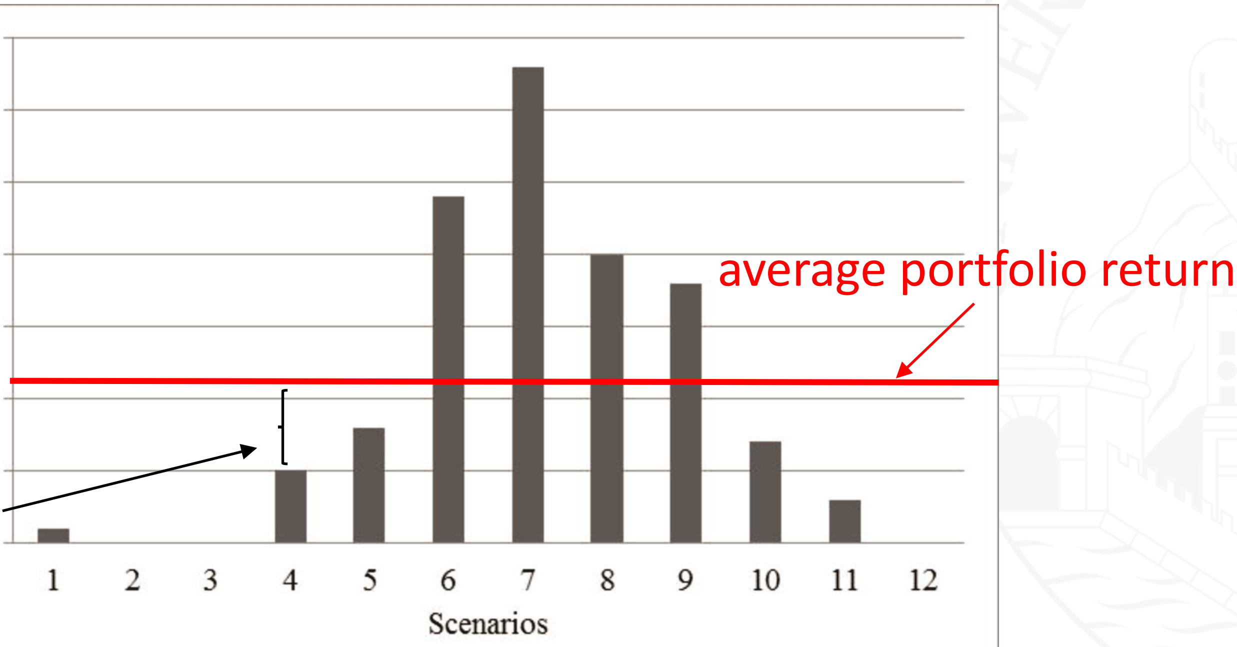
Mixed integer programming and finance

$$d_t = \left|\sum_{j=1}^{n} r_{jt}x_j - \sum_{j=1}^{n} r_j x_j\right|$$



Downside Mean Absolute Deviation

Mixed integer programming and finance

$$\min \frac{1}{T}\sum_{t=1}^{T} d_t$$

$$d_t + \sum_{j=1}^{n} a_{jt} x_j \geq 0 \qquad t = 1,\ldots,T$$

$$\sum_{j=1}^{n} r_j x_j \geq \mu_0$$

$$\sum_{j=1}^{n} x_j = 1$$

$$x_j \geq 0 \;\; j = 1,\ldots,n$$

$$d_t \geq 0 \; t = 1,\ldots,T$$

Downside Mean Absolute Deviation

$= \frac{1}{2}$ Mean Absolute Deviation

Fixed transaction costs
Limited number of assets
Transaction lots

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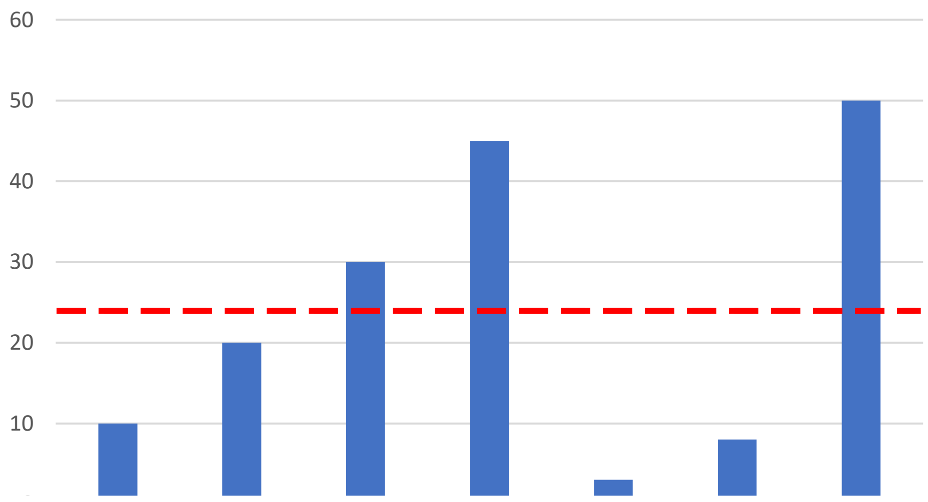
Connecting Min sum and Min max

The most used objective function is the **sum** or **average** of an individual measure (in a minimization problem, min the average over all 'agents')

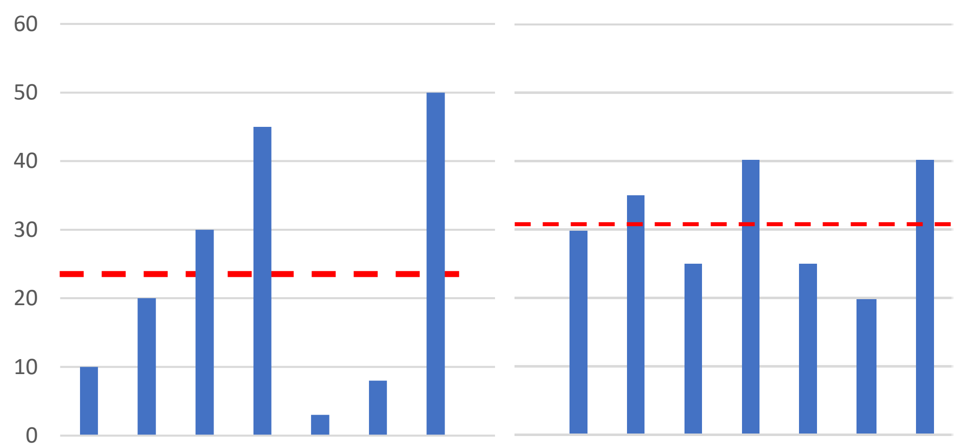
The **average** criterion does not take into account the **variability**



Connecting Min sum and Min max

The alternative is to minimize the **max** value of the individual measure

The **Min max** protects the worst case only



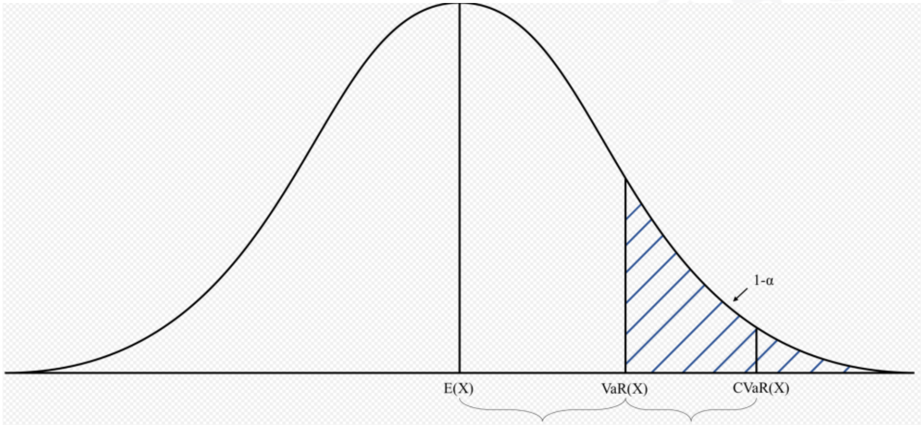
Min average

Min max

Connecting Min sum and Min max

The Conditional Value-at-Risk (CVaR): the average loss, given a confidence level

It is defined for random variables with continuous distribution



$$CVaR(X) = E(X | X \geq VaR_{\alpha}(X))$$

The concept can be adapted to a non-stochastic discrete case:

Minimize the average over a given percentage of the worst 'agents'

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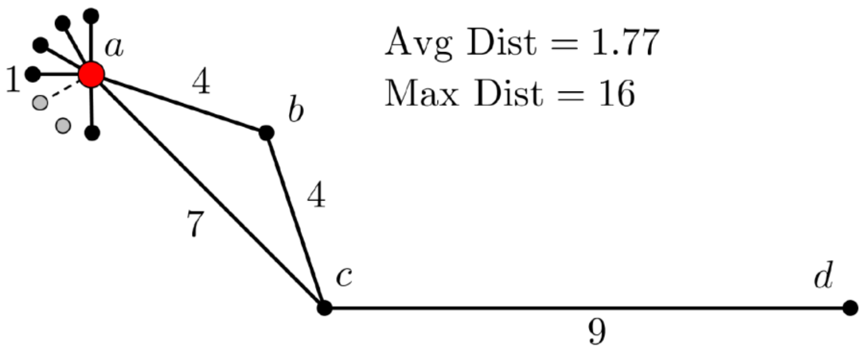
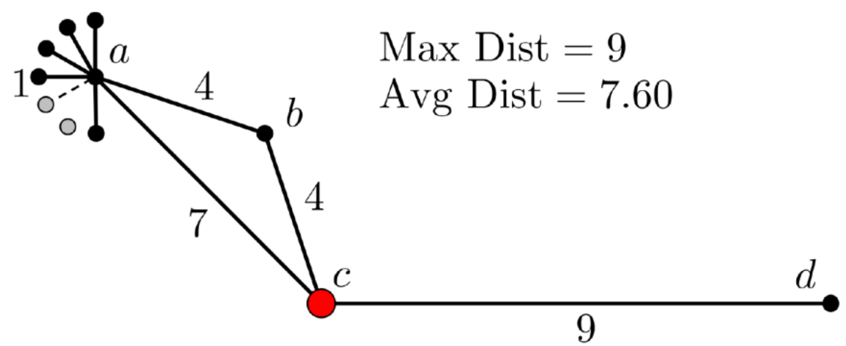
Connecting Min sum and Min max

30 customers

Min the average distance:
Location a is selected



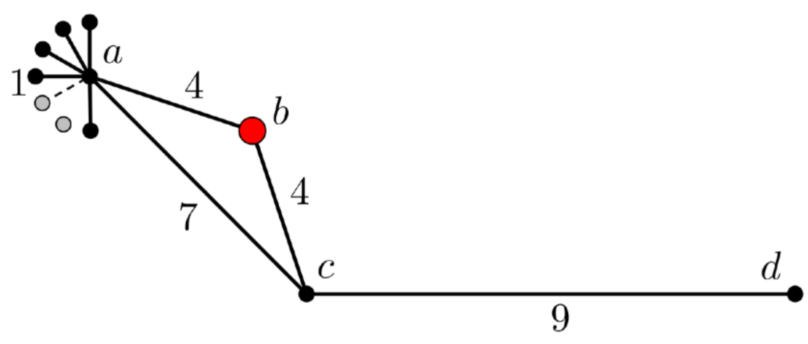
Min the maximum distance:
Location c is selected



Connecting Min sum and Min max

Minimize the average distance for the 10% worst customers

Location b is selected



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Connecting Min sum and Min max

In a minimization problem, for any α,
the Worst Conditional Average (WCA) is
the largest average over a percentage α of 'agents'

$$\begin{array}{ll} \min & c^\top x \\ \text{subject to} & Ax = b \\ & x_B \in \mathbb{Z}_+^{|B|} \\ & x_N \geq 0 \end{array}$$

Min average

$$\begin{array}{lll} \min & \lceil \alpha S \rceil u + \sum_{\ell=1}^{S} v_\ell & \\ \text{subject to} & \lceil \alpha S \rceil (u + v_\ell) \geq (c^\ell)^\top x & \ell = 1, \ldots, S \\ & Ax = b & \\ & v_\ell \geq 0 & \ell = 1, \ldots, S \\ & x_B \in \mathbb{Z}_+^{|B|}, x_N \geq 0 & \end{array}$$

Number of 'agents'

Min WCA(α)

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Data and decisions

The value of information and the role of time

Everything is known and all decisions are taken together

⇩

Deterministic models

Off-line models

Nothing is known and decisions are taken one at a time

⇩

On-line models

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Data and decisions

Competitive ratio of an on-line algorithm H (minimization problem)

$$R_H = \inf \{ r \mid H(I)/O(I) \leq r \text{ for all instances } I \}$$

optimum if all is known in advance

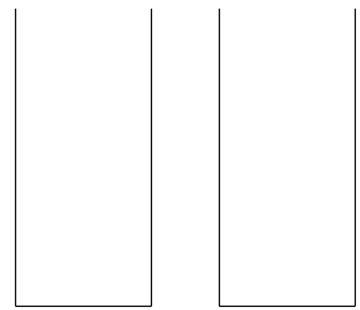
Optimality of an on-line algorithm H

$$R_H \leq R_A \text{ for any algorithm A}$$

Data and decisions

Scheduling on two parallel machines
or
Partition

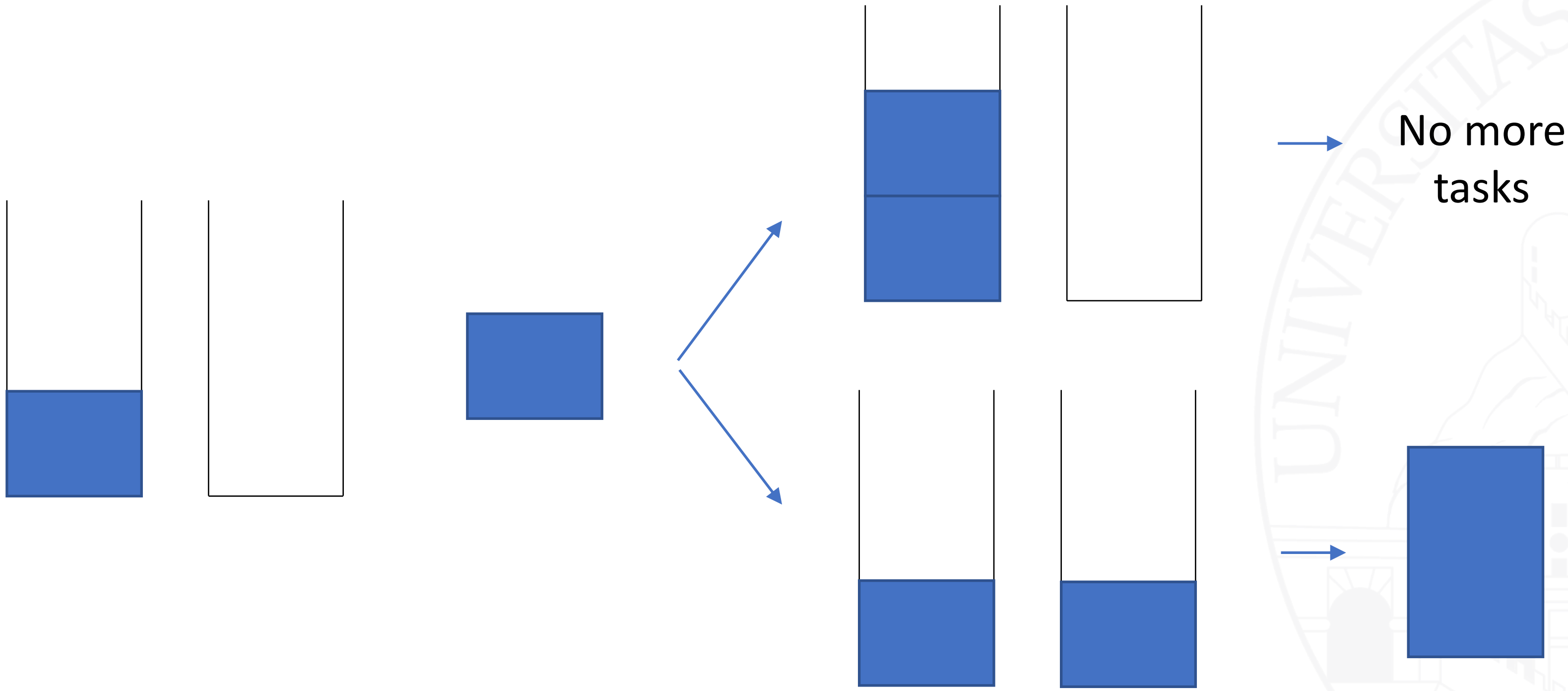
On-line problem



Tasks arrive one by one and each task must be immediately assigned to a machine

Input: two machines, tasks
Output: assignment of tasks to machines
Objective: minimization of the makespan

Data and decisions



No on-line algorithm can do better than $\frac{3}{2}$

$$R \geq \frac{3}{2}$$

H: assign incoming task to the machine with smallest load

$$\boxed{R_H \leq \frac{3}{2} \quad \text{optimal}}$$

Data and decisions

The total sum of the tasks is known in advance $R_{H2} = \frac{4}{3}$

A buffer of length k is available to maintain k tasks (k=1 is sufficient) $R_{H1} = \frac{4}{3}$

Semi-online problems

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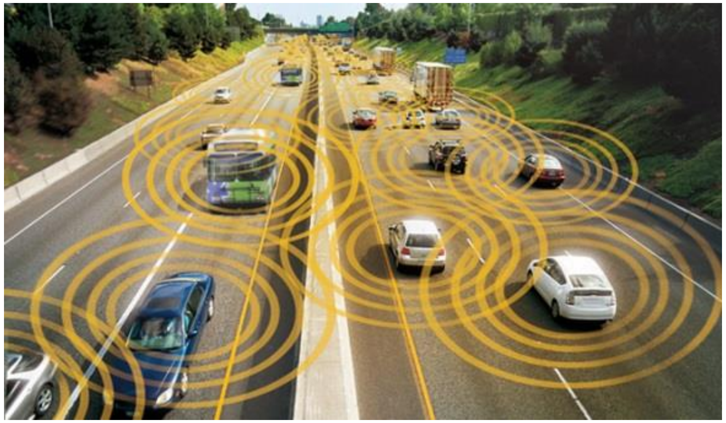
Optimization and traffic science



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Optimization and traffic science

Long paths for some drivers ← Minimum total travel time

User-equilibrium vs **System-optimum**

Selfish behaviour

(Nash equilibrium,

no one can switch to a better path) → Congestion

Price of anarchy

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Optimization and traffic science

Min total travel time

on paths of limited inconvenience

$$t_{ij}^{FF}[1 + 0.15(\frac{x_{ij}}{u_{ij}})^4]$$

Travel time on arc (i,j) with flow x_{ij}

$\Rightarrow$ Non-linear optimization problem
(with an exponential number of paths)

$\Downarrow$

Piecewise linear approximation
(heuristic generation of paths)

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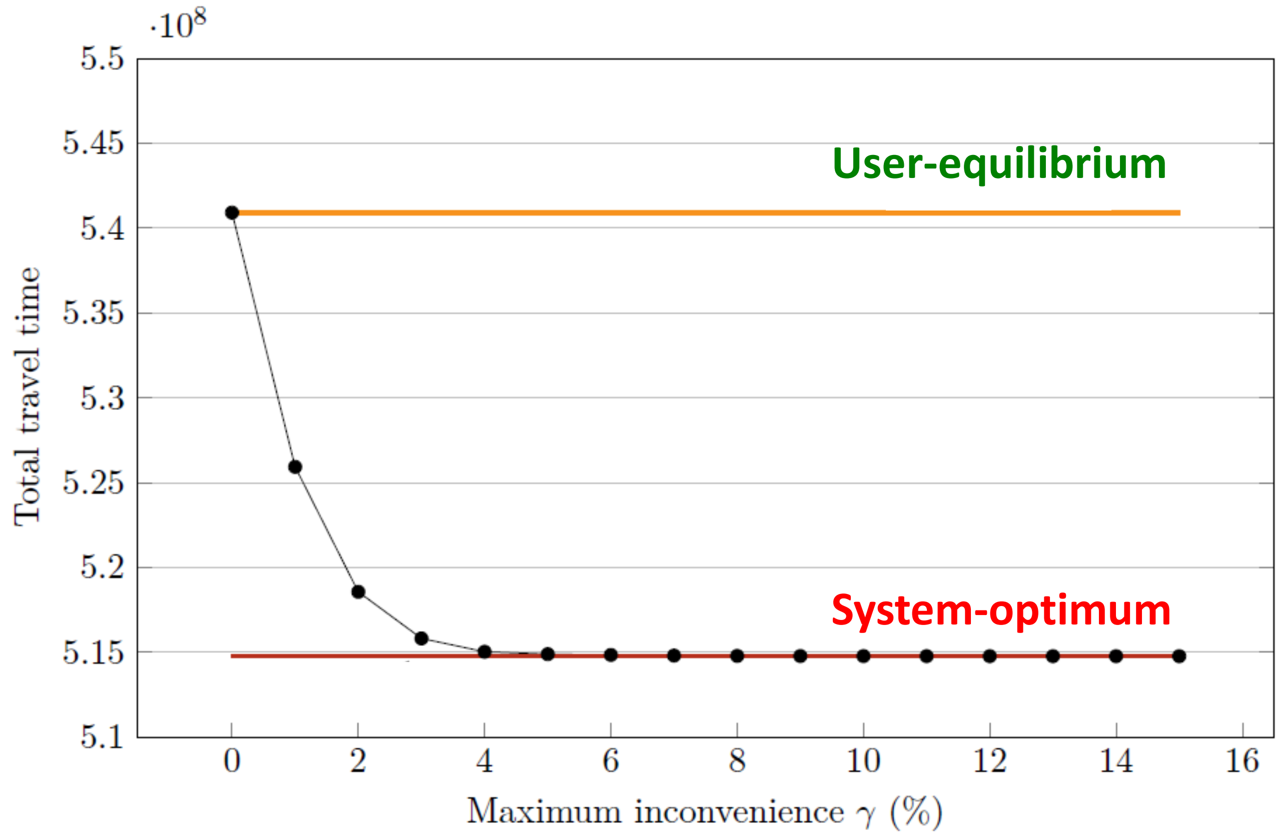
Optimization and traffic science



General solver and ad hoc heuristics

Problem ⇨ Ad hoc heuristic

⇩

MILP

⇩

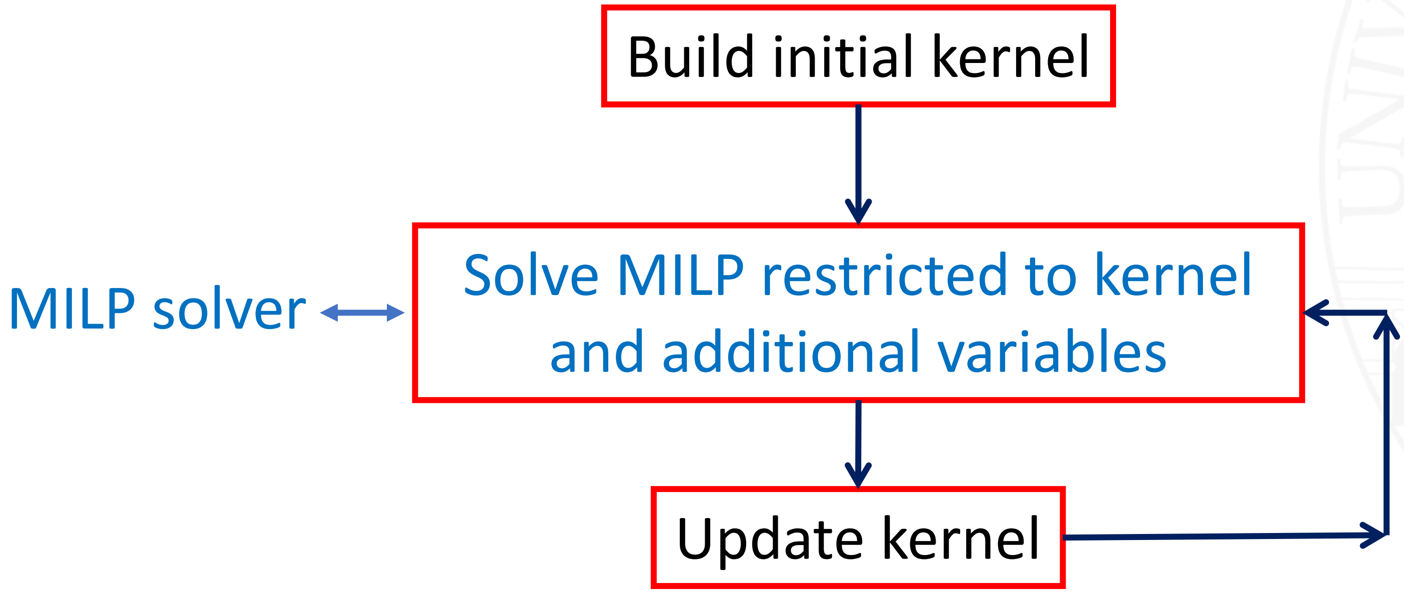
Solver

Heuristics for classes of problems?

Heuristics that use a solver?

General solver and ad hoc heuristics

Kernel search



General solver and ad hoc heuristics

Multi-dimensional knapsack

Portfolio optimization

Index tracking

Capacitated facility location

Bi-objective enhanced index tracking

Single source facility location

Mixed integer linear program

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Andrea Mor

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Lorenzo Peirano

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