Wednesday, 8:30 - 9:00

■ WA-01

Wednesday, 8:30 - 9:00 - Room: C V VI

Opening

Stream: Plenaries
Invited session
Chair: *Tibor Illés*

Wednesday, 9:00 - 10:00

■ WB-01

Wednesday, 9:00 - 10:00 - Room: C V VI

Plenary I

Stream: Plenaries
Plenary session
Chair: *Tibor Illés*

1 - Distributionally robust linear quadratic control
Daniel Kuhn

Linear-Quadratic-Gaussian (LQG) control is a fundamental control paradigm that is studied in various fields such as engineering, computer science, economics, and neuroscience. It involves controlling a system with linear dynamics and imperfect observations, subject to additive noise, with the goal of minimizing a quadratic cost function for the state and control variables. In this work, we consider a generalization of the discrete-time, finite-horizon LQG problem, where the noise distributions are unknown and belong to Wasserstein ambiguity sets centered at nominal (Gaussian) distributions. The objective is to minimize a worst-case cost across all distributions in the ambiguity set, including non-Gaussian distributions. Despite the added complexity, we prove that a control policy that is linear in the observations is optimal for this problem, as in the classic LQG problem. We propose a numerical solution method that efficiently characterizes this optimal control policy. Our method uses the Frank-Wolfe algorithm to identify the least-favorable distributions within the Wasserstein ambiguity sets and computes the controller's optimal policy using Kalman filter estimation under these distributions.

Wednesday, 10:00 - 11:30

■ WC-02

Wednesday, 10:00 - 11:30 - Room: C 103

Algorithms

Stream: Game theory
Invited session
Chair: *Giancarlo Bigi*

1 - A new nucleolus-like method to compute the priority vector of a pairwise comparison matrix
David Bartl

In multiple-criteria decision-making, given objects, such as alternatives, are often evaluated pairwise with respect to some criterion. The relative importance of the two elements in a pair is usually rated by a real number and by using the multiplicative scale; that is, the number means how many times an element is better (or more important) than the other in the pair. Then, the Geometric Mean Method (GMM) and Saaty's Eigenvector Method (EVM) are prominently used to find the priority vector of a given pairwise comparison matrix (PCM).

In this paper, we consider the more general case when the entries of the PCM are elements of a divisible alo-group (Abelian linearly ordered group), cf. the "general unified framework for pairwise comparison matrices in multicriteria methods" by Cavallo & D'Apuzzo (2009). In this case, Saaty's EVM cannot be used due to its intrinsic properties. The GMM can easily be adapted to find the priority vector of the PCM with entries from a divisible alo-group (Cavallo & D'Apuzzo, 2012) and, to our best knowledge, is the only currently known method that can be used in this setting.

Inspired by the concept of nucleolus from cooperative game theory (Schmeidler, 1969), we propose a new nucleolus-like method to compute the priority vector of a pairwise comparison matrix with entries from any divisible alo-group. The method utilizes the theory of linear programming in abstract spaces (Bartl, 2007).

2 - Optimization framework of a two-sided auction based logistical coordination platform
Márton Benedek, Kolos Ágoston

We introduce an optimization-based coordination platform to increase the efficiency of regional package delivery. The proposed mechanism is based on a two-sided auction framework, with package-type bids on the one side and vehicle/delivery-capacity type offers on the other side. After collecting all the bids for the target day, the algorithm determines the set of accepted package bids, the set of active vehicles, allocates the accepted packages to the active vehicles, and determines the route of the selected vehicles. The clearing algorithm is formulated as an optimization problem, maximizing the total value of accepted package bids, while taking into consideration various constraints related to package allocation, vehicle capacities, vehicle net income and routing considerations. The possible generalizations of the proposed model are discussed with regard to the relaxation of the vehicle-profitability constraints in the case of bidding entities with multiple vehicles, the possible alternative approaches for distributing the potential surplus and the possible consideration of vehicles with already required pickup and delivery points.

3 - Projected solutions for quasi-equilibria
Giancarlo Bigi

The concept of a projected solution for quasi-variational inequalities and generalized Nash equilibrium problems was recently introduced, due to the modeling of deregulated electricity markets where the constraint map is not a self-map. In this talk we show that the projected solutions of a quasi-equilibrium problem correspond to the classical solutions of an auxiliary quasi-equilibrium problem. This can be achieved by doubling the number of variables and adding an appropriate term. In this way algorithms for quasi-equilibria can be exploited for computing projected equilibria through the auxiliary problem. However, its structure does not guarantee the fulfilment of the assumptions required for the convergence of algorithms. In particular, descent and extragradient algorithms need that each feasible point is a fixed point for the constraint map and this not true in the case at hand. Thus, we show that under suitable assumption an ad-hoc choice of parameters allows convergence of the extragradient algorithm without the above requirement. the results of preliminary numerical tests are reported as well.

The talk is based on joint papers with Marco Castellani and Sara Latini

---

■ WC-03

Wednesday, 10:00 - 11:30 - Room: C 104

Mixed Integer Programming I

Stream: Discrete Optimization
Invited session
Chair: *Markó Horváth*

1 - LP reformulation of combinatorial optimization problems aided by combinatorial methods
Bogdan Zavalnij, Sandor Szabo

Linear Programming (LP) is a well known technique for solving different optimization problems, including combinatorial optimization problems (COP). In this latter case we use Integer Linear Programming (ILP) or binary (0-1) LP to solve the program to optimality, or use the real (or continuous) relaxation of the discrete program to establish an upper bound for a maximization problem or a lower bound for a minimization problem. Typically a COP can be reduced to a discrete LP in more than one ways. This is also true for the Maximum Clique Problem (MCP) or k-clique problem (k-CP), which we shall use to illustrate our considerations.

The known discrete LP formulations of a COP usually model the problem in some straightforward manner. The known LP formulations of the MCP based on compiling lists of non-edges, sets non-neighbors of nodes, independent sets (or their extensions) of the underlying graph.

We would like to point out, that some more involved combinatorial procedures can aid of setting up new LP reformulations. The point is, that using some fast (usually polynomial) algorithm can discover information that can be added to the LP reformulation, and thus make the program more emanable for the modern LP solvers. The auxiliary combinatorial procedures, we will use, are legal coloring and b-fold coloring. Working with the so-called dominance relations and its generalizations; applying the struction transformation will further improve the LP reformulation.

2 - Constraint Programming formulation for a real-world final exam scheduling problem with parallel sessions based on short time intervals

László Kálmán Trautsch, *Bence Kovari*

Scheduling final exams is subject to various requirements that differ by countries and universities. A range of personal, institutional, and regulatory factors should be considered at the same time for creating an optimal schedule. We propose a Constraint Programming model for scheduling final examinations at the Department of Automation and Applied Informatics, Budapest University of Technology and Economics. The requirements of this scheduling problem regard an examination period which is divided into 5-minute intervals. Heterogeneous student groups and instructors with various roles are scheduled to parallel sessions based on these time intervals. The cost function is defined by multiple types of constraints, such as balancing the workload, optimizing start and end times of sessions and breaks, and penalizing leaks in the schedules of instructors. We present the formulation of the various complex requirements of final exam scheduling and demonstrate the results of applying a Constraint Programming solver on our model to find feasible solutions for a real-world scheduling problem involving 101 students.

3 - Maximal Hamming packing search: contact graph-based model MILP improvements

Péter Naszvadi, *Mátyás Koniorczyk*

We consider mixed Hamming packings: maximal subsets of a given Hamming space over a mixed alphabet keeping that every selected codeword pair has a minimum distance, using mixed integer programming models. There are many known and efficient approaches to the problem in the literature, including clique-reformulation, specific exhaustive search algorithms, etc. Our contribution is based on mixed integer programming which was not frequently used before as it was not competitive with other methods. We overcome this issue by introducing a reduction technique and further constraints based on structural properties. In particular, we introduce a presolving column reduction technique based on our idea of adopting the notion of contact graphs motivated by the Tammes-problem. We prove that for every mixed Hamming-space over alphabets having minimal cardinality 3, there is a maximal Hamming-packing in them with a connected contact graph with minimal vertex degree at least 2. As a corollary of this lemma we can introduce a new type of linear constraints for the MILP model, which significantly simplify the solution. We present particular results for a number of particular instances: in case of binary-ternary problems we obtain best known results in competitive computational time.

■ WC-04

Wednesday, 10:00 - 11:30 - Room: C105

Large scale optimization and applications 1

Stream: Large scale optimization and applications

Invited session

Chair: *Nataša Krklec Jerinkić*

1 - Spectral Stochastic Gradient Method with Additional Sampling for Finite and Infinite Sums

Nataša Krklec Jerinkić, *Valeria Ruggiero*, *Ilaria Trombini*

In this paper, we propose a new stochastic gradient method for numerical minimization of finite sums and its modified version applicable to more general problems where the objective function is in the form of mathematical expectation. The method is based on a strategy to exploit the effectiveness of the well-known BB-like rules for updating the step length in the standard gradient method. The proposed method adapts the aforementioned strategy into the stochastic framework by exploiting the same SAA estimator of the objective function for several iterations. Furthermore, the sample size is controlled by an additional sampling which also plays a role in accepting the proposed iterate point. Moreover, the number of "inner" iterations with the same sample is also controlled by an adaptive rule which prevents the method from getting stacked with the same estimator for too long. Convergence results are discussed for the finite and infinite sum version, for general and strongly convex objective functions. Numerical experiments on well-known datasets for binary classifications show very promising performance of the method, without the need to provide special values for hyperparameters on which the method depends.

2 - A Simple Stochastic Trust-Region Method for Training Neural Network Classification Models

Mahsa Yousefi, *Stefania Bellavia*, *Benedetta Morini*

Deep learning (DL), employing deep neural networks, is widely used in tasks like image recognition. Training these networks involves solving finite-sum minimization problems. Due to the impracticality of computing true gradients or Hessians for large-scale DL problems, stochastic methods with subsampling are typically employed. Stochastic gradient descent (SGD) and its variations are popular in deep learning due to their simplicity and low per-iteration cost. Second-order methods are also explored for their potential to utilize curvature information, aiding navigation of complex landscapes efficiently. However, using second-order information entails the computational cost of computing Hessians or their approximations. To combine the advantages of both first- and second-order methods, we introduce a stochastic approach based on a simple trust-region model that utilizes approximate partial second-order curvature information efficiently. We provide convergence assurances and empirical assessments of this method in image classification tasks.

3 - Neural Network Models for Eigenvalue Problems

Marko Hajba, *Luka Grubisic*

We study eigenmode localization for a class of elliptic reaction-diffusion operators. As the prototype model problem we use a family of Schrödinger Hamiltonians parametrized by random potentials. Our computational model is posed in the truncated finite domain, and this is an approximation of the standard Schrödinger Hamiltonian. Our chosen task is to compute localized bounded states at the lower end of the spectrum. Neural networks (NNs) are used as surrogate models which represent dependence of the ground state or landscape function on the localizing potential, depending on a problem we are solving. Further, we will also demonstrate the use of Variational Physics Informed Neural Network, together with the residual type error estimates, to obtain the ground state of the eigenvalue problem. Error estimators will be introduced to monitor the performance of the model. We present a host of numerical experiments to benchmark the accuracy and performance of the proposed algorithms.

Wednesday, 12:00 - 13:30

■ WD-02

Wednesday, 12:00 - 13:30 - Room: C 103

Applications

Stream: Game theory
Invited session
Chair: *Attila Tasnádi*

1 - Limitations of the MedRank algorithm
Attila Tasnádi
Voting rules can be derived as distance minimization problems. Under quite restrictive conditions the MedRank algorithm minimizes the so-called Spearman footrule. We highlight the limitation of this result and also investigate the possibility of appropriate refinements of the MedRank algorithm. In addition, we show that the analogous problem does not arise when minimizing Spearman rank correlation, which results in the Borda count.

2 - Voting power in the Council of the European Union: A sensitivity analysis
Laszlo Csato, Dora Greta Petroczy
The Council of the European Union (EU) is one of the main decision-making bodies of the EU. Many decisions require a qualified majority: the support of 55% of the member states (currently 15) that represent at least 65% of the total population. We investigate how the power distribution, based on the Shapley-Shubik index, and the proportion of winning coalitions change if these criteria are modified within reasonable bounds. The influence of the two countries with about 4% of the total population each is found to be almost flat. The level of decisiveness decreases if the population criterion is above 68% or the states criterion is at least 17. The proportion of winning coalitions can be increased from 13.2% to 20.8% (30.1%) such that the maximal relative change in the Shapley-Shubik indices remains below 3.5% (5.5%). Our results are indispensable to evaluate any proposal for reforming the qualified majority voting system.

3 - Convergence of a Two-Player Version of Macqueen's k-means Algorithm
Stéphan Sémirat
We extend Macqueen's version of the k-means algorithm (Macqueen, 1967) by assuming that the algorithm involves two players. Player 1, the sender, equipped with its type, emits data at no cost, and Player 2, the receiver, equipped with Player 1's data, chooses an action. In particular, data are pooled with regard to the action they are associated with. Pure perfect Bayesian equilibria (PBE) in this game are characterized by partitions of the types, in which no type prefers to be pooled with types it is not currently pooled with, given the decisions of the receiver, optimally chosen at each cell of the partition. In this context, the inductive step of the k-means algorithm proceeds as follows: (i) the sender is in charge of moving a type from its cell to a cell it prefers; (ii) the receiver readjusts the cell-contingent actions accordingly. Since players might have different objective functions, the simple convergence argument invoked in Macqueen's context (monotonicity of stages (i) and (ii) with regard to the intra or inter variances) no more holds. Instead, we use a combinatorial argument to achieve convergence, for every initial partition and every path taken by the algorithm. We do this for utility functions that are single-peaked and single-crossing, with an upward bias for the sender. The family covers, but goes much beyond, the usual Euclidean distance. However, our argument crucially relies on the order on types and actions.

■ WD-03

Wednesday, 12:00 - 13:30 - Room: C 104

Mixed Integer Programming II

Stream: Discrete Optimization
Invited session
Chair: *Ambros Gleixner*

1 - Complex geometrical test for optimality conditions in Interval Branch and Bound method
Mihály Gencsi, Boglárka G.-Tóth
This study focuses on solving constrained nonlinear programming problems. The Interval Branch and Bound (IBB) method is the most widely used approach for obtaining rigorous solutions. However, few IBB implementations utilize the Karush-Kuhn-Tucker or Fritz-John optimality conditions to eliminate non-optimal boxes. The Fritz-John optimality condition involves solving an interval-valued system of equations. When solving these equations, we sometimes cannot reduce the size of the box due to overestimation of the gradient boxes.

This study considers the optimality conditions from a geometric perspective. A new, more complex geometrical optimality test is introduced to precede the Fritz-John optimality condition. The goal is to speed up the IBB method and eliminate unnecessary computations.

The test easily ensures that there is no local optimum in the box, or that we cannot reduce the size of the box by solving the optimality condition system because of the overestimation of the gradient boxes. We will describe the geometric test and its usefulness, as well as present experimental results demonstrating the effectiveness of the complex geometrical test.

2 - Piecewise linear modeling of head-dependent hydropower function on non-grid triangulation
Peter Dobrovoczki, Tamas Kis
Modeling piecewise linear functions with mixed integer linear programming is a challenging problem with several applications. We present a novel method to construct efficient MILP representation of piecewise linear functions of two variable over non-grid triangulations. Our method consists of the construction of a novel heuristic to find biclique cover to derive an ideal formulation of combinatorial disjunctive constraints that are pairwise independent branching (IB) scheme representable. We extend our method for the case when the combinatorial disjunction is not IB-representable by coloring an auxiliary graph. We apply our techniques for constructing a piecewise linear approximation of 2-variable (non-linear) functions. Firstly, we propose a fast heuristic algorithm to construct a piecewise linear approximation of a function over a bounded domain such that the approximation error is low in expectation. We apply our methods to model the head-dependent hydropower function in a hydropower plant scheduling application.

3 - A proof system for certifying symmetry and optimality reasoning in integer programming
Ambros Gleixner, Jasper van Doornmalen, Christopher Hojny, Leon Eifler

We present a proof system for establishing the correctness of results produced by optimization algorithms, with a focus on mixed-integer programming (MIP). Our system generalizes the seminal work of Bogaerts, Gocht, McCreesh, and Nordström (2022) for binary programs to handle any additional difficulties arising from unbounded and continuous variables, and covers a broad range of solving techniques, including symmetry handling, cutting planes, and presolving reductions. Consistency across all decisions that affect the feasible region is achieved by a pair of transitive relations on the set of solutions, which relies on the newly introduced notion of consistent branching trees. Combined with a series of machine-verifiable derivation rules, the resulting framework offers practical solutions to enhance the trust in integer programming as a methodology for applications where reliability and correctness are key.

■ WD-04

Wednesday, 12:00 - 13:30 - Room: C105

Large scale optimization and applications 2

Stream: Large scale optimization and applications
Invited session
Chair: *Greta Malaspina*

1 - Probabilistic Trust Region Method for solving Multi-Objective Problems
Luka Rutešić, Natasa Krejic, Nataša Krklec Jerinkić

The problem considered is a multi-objective optimization problem, in which the goal is to find an optimal value of a vector function representing various criteria. An algorithm which utilizes trust region framework with probabilistic model functions able to cope with noisy problems and approximate functions and their derivatives is derived and analysed. We prove the almost sure convergence of the proposed algorithm to a Pareto critical point if the model functions are good approximations in probabilistic sense. Numerical results demonstrate effectiveness of the probabilistic trust region by comparing it to competitive stochastic multi-objective solvers. The application in supervised machine learning is showcased by training non discriminatory Logistic Regression models on different size data groups. Additionally, we use several test examples with irregularly shaped fronts to exibit the efficiency of the algorithm.

2 - Evolving relocation rules for the Container Relocation Problem using Genetic programming
Mateja Đumić

More than 90% of global trade transportation operates by sea, and over 15% is done using containers. Consequently, container transportation plays a significant role in global trade. Containers are stored in container yards while waiting to be loaded onto ships. Container yards have limited capacity, so containers are usually stored in stacks on each other. In most cases, the sequence in which these containers must be loaded onto the ship is unknown, making it impossible to rearrange them so that each container can be retrieved without relocating another container blocking it. The Container Relocation Problem (CRP) is a combinatorial optimization problem tasked with finding a sequence of container relocations to retrieve all containers in a defined order while optimizing one or more criteria. CRP is an NP-hard problem, meaning that, in most cases, it can not be solved exactly. Because of that, heuristic approaches are generally used to solve it. The simplest heuristic methods for solving CRP are Relocation Rules (RRs). RRs are fast and simple, but their creation requires domain knowledge and must be developed for each criterion separately. Within this study, the process of developing RRs will be automated using Genetic Programming (GP) to overcome this problem. Experimental results show that RRs developed using GP achieve better results than existing manually developed rules and have good generalization ability, which makes this approach a viable option.

3 - Randomized Gauss-Newton methods for large scale nonlinear least squares
Greta Malaspina, Stefania Bellavia, Benedetta Morini

We address the solution of large-scale nonlinear least-squares problems by stochastic Gauss-Newton methods combined with a line-search strategy. The algorithms proposed have computational complexity lower than classical deterministic methods due to the employment of random models inspired by randomized linear algebra tools. Under suitable assumptions, results on the ability to achieve a desired level of accuracy in the first-order optimality condition can be established. We discuss the construction of the random models, the iteration complexity results to drive the gradient below a prescribed accuracy and present results from our computational experience.

Wednesday, 14:45 - 16:15

■ WE-02

Wednesday, 14:45 - 16:15 - Room: C 103

Combinatorial Optimization I

Stream: Discrete Optimization
Invited session
Chair: *Márton Benedek*

1 - Passing the Limits of Pure Local Search for Weighted k-Set Packing
Meike Neuwohner

Given a collection S of sets, each of cardinality at most k, and equipped with positive weights, the weighted k-Set Packing problem asks for a disjoint subcollection of S of maximum total weight. The case k = 2 is equivalent to weighted matching and can thus be solved in polynomial time. For k >= 3, already the unweighted k-Set Packing problem, the special case where all weights are equal to 1, is NP-hard.

The technique that has proven most successful in designing approximation algorithms for both the unweighted and the weighted k-Set Packing problem is local search. For the unweighted problem, the state-of-the-art is a polynomial-time (k+1)/3+epsilon-approximation by Fürer and Yu. For general weights, we recently obtained a polynomial-time (k+epsilon_k)/2-approximation, where epsilon_k converges to 0 as k approaches infinity. We further proved that for general weights, one cannot achieve a better guarantee than k/2 by only considering local improvements of logarithmically bounded size.

In this talk, we show how to breach the k/2 barrier by employing a black box algorithm for the unweighted k-Set Packing problem to generate local improvements of super-logarithmic size. In doing so, we obtain a polynomial-time 0.4986*k+0.5194-approximation algorithm for weighted k-Set Packing. Our techniques further allow us to establish a general connection between the approximation guarantees achievable for unit and general weights.

2 - Optimizing schedules by the aspect of fairness
Martin Rónai-Kovács, *Szilvia Jáhn-Erdős*

Nowadays, schedules appear at many points in our lives, and their fairness is increasingly important. This view is reinforced when we think of schedules not just as our daily schedules, our work assignments or our public transport timetables, but also as the decision support systems that help us, for example, in a loan decision. In all these cases, it is important to base solutions only on information that is relevant when making the schedule, and it is also important to be able to point out when an unfair decision is made.

Given the diversity of scheduling problems, it requires great care to create a computation method that works for all problems. One such is entity-based fairness, which can abstract away schedules sufficiently to objectively determine the degree of fairness. However, if we want to improve the schedule, we need to consider how to use this result.

The primary question to be examined is when a change in the schedule counts as an improvement, and to what extent it results in an improvement. In addition, I examine whether an optimal improvement can be identified in each case and what the necessary conditions for this are. I define a method to propose a change to the schedule.

■ WE-03

Wednesday, 14:45 - 16:15 - Room: C 104

Interior-Point Methods for Linear Complementarity Problems I

Stream: Advances in theory and practice of interior-point methods
Invited session
Chair: *Petra Renáta Rigó*

1 - New class of algebraically equivalent transformations for predictor-corrector interior-point algorithms
Petra Renáta Rigó, *Tibor Illés*, *Roland Török*

In this talk we present predictor-corrector (PC) interior-point algorithms (IPAs) for solving sufficient linear complementarity problems. We use the algebraic equivalent transformation (AET) technique in order to define the search directions. We give a unified complexity analysis of the PC IPAs by using a whole class of AET functions. We show that the PC IPA using any member of the new class of AET functions has polynomial iteration complexity in the handicap of the problem's matrix, the size of the problem, the starting point's duality gap and in the accuracy parameter.

2 - Implementation of predictor-corrector interior-point algorithms for sufficient linear complementarity problems
Roland Török, *Tibor Illés*, *Petra Renáta Rigó*

We generated new problems for sufficient linear complementarity problems. In this talk we present the obtained numerical results based on the implementation of predictor-corrector interior-point algorithms. For the determination of search directions, we used a new class of algebraically equivalent transformation functions and a kernel function, as well. We compared the results for the algorithms using different search directions.

3 - New class of algebraically equivalent transformations for predictor-corrector algorithms solving symmetric cone horizontal linear complementarity problems
Zsolt Darvay, *Petra Renáta Rigó*

We generalize a generic predictor-corrector (PC) interior-point algorithm (IPA) solving horizontal linear complementarity problems (LCPs) over Cartesian product of symmetric cones. We propose the first class of algebraically equivalent transformation (AET) functions for PC IPAs to horizontal LCPs over Cartesian product of symmetric cones. For the first time we use two different AET functions in the predictor and corrector steps, respectively.

■ WE-04

Wednesday, 14:45 - 16:15 - Room: C105

P-graph Algorithms

Stream: P-graph algorithms and applications
Invited session
Chair: *Marton Frits*

1 - Direct Calculation: A Novel P-graph Based Method for Determining Reliability

Mihály István Sümegi, Ákos Orosz

Reliability is a key indicator when designing complex industrial processes, consequently, methods for its determination have already been published in the middle of the last century. Most deterministic methods are based on recognising either specific structural patterns or smaller subsystems that guarantee operation or failure.

This paper explores a novel approach which begins at the starting points of a process and iteratively calculates internal reliabilities via propagation. As a result the method does not depend on preliminary identification of structural patterns but is still deterministic and provides an exact result.

The method is based on the P-graph framework, which is a combinatorial tool for process synthesis. The framework's unambiguous representation of complex processes provides the basis for the correct propagation. Through the logical connections represented by the bipartite graph, the necessary operations at each step can be identified.

Several case studies have been examined to compare the proposed approach to previous ones. The determined reliability values match the results of previous algorithms even for the most complicated networks. As expected, the efficiency of the method highly depends on the structure of the process, however, the tests indicate that it can significantly surpass previous methods for a large number of cases.

2 - Extension of P-graph Framework to Simultaneously Cover Directed and Nondirected Elements

Ákos Orosz, Ferenc Friedler

P-graph framework has been developed to provide structure oriented mathematical basis for process network synthesis. During the more than three decades since its first appearance, it has been extended with various capabilities, and has been applied to several real-world case studies. These applications include e.g., transportation networks, reaction network synthesis, and processing systems. In some of the applications, reversible operations may appear. In the previous works, these cases are modeled as two distinct options, representing the two possible directions of operations. However, since only one of the two directions can appear in the synthesized process, additional algorithmic constraints were required. The current work proposes the extension of the P-graph framework with non-directed operations, that appear in the model as a single operating unit, and selecting their direction can be handled implicitly by the combinatorial algorithms, without the need of additional constraints. The work presents the extended formalism and the modified algorithms for problems where some or all operations are reversible. The methods are demonstrated through case studies and the results are compared with the traditional P-graph-based methods.

3 - Combining the multi-periodic and flexible input modeling techniques in the P-Graph framework

András Éles, István Heckl

This works focuses on how two major modeling techniques in the P-Graph framework can be applied together. Multi-periodic models allow to address scenarios where input feeds and/or output demands are fluctuating between time periods, and intermediate storage is not negligible. The flexible input scheme can be used when the composition of multiple inputs is arbitrary, but can be subject to custom constraints. Both techniques use multiple operating unit nodes to model complex operations. In our current work, we compare the two techniques and investigate how these should be used in a single case study. As a demonstration, a processed food production problem is solved. This problem involves multiple time periods and flexible inputs in each period, which suggests that the two modeling techniques should be integrated.

Wednesday, 16:45 - 18:15

■ WF-02

Wednesday, 16:45 - 18:15 - Room: C 103

Combinatorial Optimization II

Stream: Discrete Optimization
Invited session
Chair: *Lilla Tothmeresz*

1 - Problems on Group-labeled Matroid Bases
Tamás Schwarcz, Florian Hörsch, András Imolay, Ryuhei Mizutani, Taihei Oki

Several combinatorial optimization problems involve additional constraints, such as parity, congruency, and exact-weight constraints. These constraints are subsumed by group-label constraints defined as follows: given a labeling of a ground set to an abelian group and a prescribed set F of forbidden labels, we define a subset of the ground set F-avoiding if the sum of the labels of its elements is not in F. Non-zero and zero problems are particularly important special cases, where F consists of the zero and non-zero labels, respectively.

We study the problems of finding F-avoiding bases and common bases of two matroids. For the single matroid case, finding an F-avoiding basis is hard if F is part of the input, while for a fixed |F|, we show the polynomial solvability of the problem in several cases, including matroids representable over fixed, finite fields. Finding a zero common basis of two matroids is hard for any nontrivial group, while we show that the solvability of finding a non-zero basis depends on the group. Furthermore, if the group is finite, we give a randomized algebraic algorithm for finding an F-avoiding common basis of two matroids represented over the same field, with running time polynomial in |F| and the group size.

2 - On the selection of an initial set of conditions for submodular function maximization for fully connected graph instances
Eszter Csokas, Tamas Vinko

The submodular function maximization (SFM) problem is a popular and well-researched combinatorial optimization problem. Nemhauser and Wolsey published the well-known constraint generating (CG) algorithm in 1981. CG solves a reduced MIP problem first, which is extended iteratively with additional constraints. In that paper, Nemhauser and Wolsey stated also that four problems are fundamental to solving integer programming models. One of them is the set of initial constraints, i.e., how to choose this set correctly, efficiently. Since the greedy method provides the approximation (1-1/e), it is traditionally used as the starting point for CG-type solving algorithms. In our previous work, we have given a centrality metric that can be used to define an initial set of constraints for SFM tasks with non-fully bipartite graph representation. It turns out that choosing a starting point different from the greedy solution can provide a more efficient solution in terms of runtime for CG-type solver algorithms. The focus of our current research is to extend the above mentioned centrality metric for SFM tasks with fully-connected bipartite graph representation. The effectiveness of using the starting point proposed by the centrality metric will be demonstrated.

3 - Relationships between the geometry of graph polytopes and graph structure
Lilla Tothmeresz, Tamas Kalman

The symmetric edge polytope is a lattice polytope associated to a graph, that is actively investigated both because of its beautiful geometry, and also because of its connections to applications. One of these applications is the Kuramoto synchronization model of physics, investigating interacting oscillators, whose interactions are modelled by a graph. By Charney, Chen and Davies, an upper bound on the number of steady states can be obtained from the volume of the symmetric edge polytope. One can also investigate „non-symmetric" edge polytopes, that are assigned to directed graphs instead of undirected ones. Moreover, one can even generalize them to regular (oriented) matroids. For these more general polytopes, many interesting geometric phenomena uncover themselves that are hidden for symmetric edge polytopes. We would like to demonstrate that the geometric properties of (graph and matroid) edge polytopes are connected to deep graph/matroid-theoretic properties. In particular, we show that the co-degree of an edge polytope is equal to the minimal cardinality of a dijoin. Other notions of combinatoral optimization also turn up naturally with respect to edge polytopes. In particular, for an Eulerian directed graph, the complements of arborescences rooted at a given vertex form a triangulation of the edge polytope of the cographic matroid. It would be nice to see if matroid duality has other interesting consequences for these polytopes.

■ WF-03

Wednesday, 16:45 - 18:15 - Room: C 104

Interior-Point Methods for Linear Complementarity Problems II

Stream: Advances in theory and practice of interior-point methods
Invited session
Chair: *Anita Varga*

1 - Bounding the handicap of a matrix
Marianna E.-Nagy, Tibor Illés, Laszlo Vegh

The linear complementarity problem (LCP), without any assumption on its coefficient matrix, is NP-hard. However, in special cases, for example, when the matrix is sufficient, the LCP is solvable efficiently. In this case, an interior point algorithm can provide a solution in polynomial time. Nevertheless, the method's theoretical complexity is directly tied to a matrix parameter known as the "handicap." Therefore an important question to address is the possible magnitude of the handicap. It is known that the handicap of a sufficient matrix is finite, but it can be exponential in the dimension of the matrix. We demonstrate that, with a fixed matrix dimension, the handicap cannot be arbitrarily large, specifically, we prove the conjecture that it cannot be double exponential in the bit size of the matrix.

2 - An Ai-Zhang-type interior-point framework for linear complementarity problems
Anita Varga, Marianna E.-Nagy

Based on their step size, interior point algorithms (IPAs) can be categorized into short- and long-step methods. Despite generally achieving superior theoretical complexity, short-step variants, in practice, are outperformed by long-step IPAs. The first long-step IPA with the same iteration complexity as the short-step variants was proposed by Ai and Zhang in 2005. To determine new search directions for IPAs, Darvay introduced the algebraically equivalent transformation (AET) technique in 2002. His main concept involved applying an invertible and continuously differentiable transformation function to the central path problem. This presentation investigates a long-step interior-point framework and a related function class for solving linear complementarity problems (LCPs). The main question we addressed is what type of functions Darvay's technique can be applied with so that an Ai-Zhang type IPA's desired convergence and complexity properties can be proved. In 2014, Potra proposed a procedure for determining the step lengths in an Ai-Zhang-type IPA for LCPs without prior knowledge of the coefficient matrix's handicap value while preserving the IPA's best known iteration complexity. We generalized this methodology for our framework applying the AET technique. In the talk, we present our numerical results to examine the efficiency of the generalized procedure for different transformation functions.

3 - Simplified Analysis of Kernel-Based Interior-Point Methods for Linear Complementarity Problems
Goran Lesaja, Zsolt Darvay, Marianna E.-Nagy, Petra Renáta Rigó, Anita Varga

We consider kernel-based Interior-point methods (IPMs) for $P_*(kappa)$- Linear Complementarity Problems (LCP) that are based on the class of Eligible kernel functions (EKFs). The importance of kernel-based IPMs stems from the fact that the iteration bounds of large-step IPMs is significantly improved for some instances of EKFs. However, the derivation of the iteration bounds for particular EKFs is usually long and quite involved which was the motivation to investigate whether this process can be simplified and under what conditions.

Hence, we introduce additional conditions on the class of EKFs, which are not very restrictive, however, they allow for the significant simplification of the analysis and calculation of iteration bounds. We derive a new simplified scheme to calculate iteration bounds and illustrate it with calculation of iteration bounds of most EKFs with polynomial and exponential barrier terms mentioned in the literature. In all cases we match the complexity obtained using the classical scheme.

■ WF-04

Wednesday, 16:45 - 18:15 - Room: C105

P-graph Applications I.

Stream: P-graph algorithms and applications
Invited session
Chair: *Zsolt Ercsey*

1 - Optimal Trajectory and Route Planning for Free Navigation of Automated Guided Vehicles
Marton Frits, Botond Bertok

In traditional industry setups automated guided vehicles (AGV) follows trajectories planned together with the layout of the storage or production facility and supported by fixed markers in the floor or on the walls. In contrast, developments in navigation techniques and the advanced computing, sensor, and communication capabilities of resent AGV makes their free movement safe and manageable. However, fleet management in a cooperative and adaptive working environment requires fast optimization algorithms to calculate and optimal movement. A two level optimization method is to be proposed herein providing a complete solution for integrated planning of optimal trajectories and routes of AGV's. Trajectory planing aims at minimizing the accelerations, i.e., forces on the vehicle and its cargo while safely reaching its target location in time from its starting location. Due to response time critical computation the multidimensional space, position, speed and acceleration are modeled by their linear approximation with proper accuracy. Route planning is computed on a graph representing interconnections of rooms and corridors. Routes are synthesized as process networks where traffic rules are taken into account as logical constraint. Optimal and alternative routes are calculated by P-graph algorithms involving several logical implications accelerating the search. The overall optimization method is to be illustrated by case studies.

2 - Learning Path Optimization by P-graph Algorithms for Curriculum Development in Higher Education
Anikó Zseni, Botond Bertok, András Horváth, Zsolt Kovács

The rapidly changing environment and the expectations set by new generations necessitate frequent revision of the curriculum in higher education. Curriculum development focuses mainly on organizing learning activities to achieve desired outcomes of educational programs. As a result, the curriculum is a roadmap from previously available competencies to the achieved target competency levels, through a series of activities that support the path. Thus, a verifiable systematic method is needed for sustainable development. In the paper all the above aspects are to be addressed by the Process Network Synthesis.

Process Network Synthesis or PNS aims at achieving all the specified desired targets by a combination of potential activities while utilizing a selection of the available resources. The P-graph framework was introduced for PNS by Friedler et al. in the early 90's. The framework involves mathematical formulation, graphical representation, and a set of combinatorial algorithms for generating the best, N-best, or all the feasible process networks for a PNS problem.

Desired competences at the end of the educational program are modeled as process targets; initially available competences, credits as resources; and each courses as a potential activity. Personal preferences and learning strategies leading to different learning paths are generated by P-graph algorithms. The results help the validation of alternative curriculum development plans.

3 - Comparison of PNS and TCPNS formulations of production scheduling for furniture manufacturing
Károly Kalauz, Botond Bertok, Marton Frits

Effective production scheduling is crucial for optimizing productivity and meeting demand in the dynamic environment of production systems. This study investigates and compares Process Network Synthesis (PNS) and Time-Constrained Process Network Synthesis (TCPNS), to assess their performance in the context of mass production. Both proposed P-graph based modeling techniques handle complex and flexible recipes. Precedence based resource scheduling model by TCPNS computes each schedule with any precision and handles complex changeover times but computation slows down due with the number of batches. Discrete Time Process Flow Modell by PNS may avoid very strict schedules due to time discretization but utilize combinatorial model reduction in the time horizon by algorithm MSG and computation is very fast regardless the huge model size. This comparative analysis highlights the strengths and considerations of each modeling approach, providing insights into their applicability and computational efficiency under varying real-life furniture manufacturing scenarios.

Thursday, 9:00 - 10:00

■ TA-01

Thursday, 9:00 - 10:00 - Room: C V VI

Plenary II

Stream: Plenaries
Plenary session
Chair: *Botond Bertok*

1 - Linear programming bounds for problems in discrete geometry
Mate Matolcsi

Let us colour the points of the plane in such a way that the endpoints of any segment of length 1 have different colours. The least number of colours needed is called the chromatic number of the unit distance graph of the plane, and determining this number is the famous Hadwiger-Nelson problem. A linear relaxation of the problem arises if we allow "fractional colourings". In this talk we will review some recent results concerning the fractional chromatic number of the plane, and the density of subsets avoiding the unit distance. In all results the theoretical considerations yield linear programs, whose solution gives bounds on the relevant quantities. A computer search is then utilized to select the best possible parameters of the linear program to obtain the sharpest possible bounds.

Thursday, 10:00 - 11:30

■ TB-02

Thursday, 10:00 - 11:30 - Room: C 103

Strategic games

Stream: Game theory
Invited session
Chair: *Miklós Pintér*

1 - Value-Positivity for Matrix Games
Raimundo Saona

Matrix games are the most basic problem in Game Theory, but robustness to small perturbations is not yet fully understood. A perturbed matrix game is one where the entries depend on a parameter which varies smoothly around zero. We introduce two new concepts: (a) value-positivity if, for every sufficiently small error, there is a strategy that guarantees the value of the error-free matrix game; and (b) uniform value-positivity if there exists a fixed strategy that guarantees, for every sufficiently small error, the value of the error-free matrix game. While the first concept captures the dependency of optimal strategies to small perturbations, the second naturally arises where the data is uncertain and a strategy should remain optimal despite that uncertainty. In this paper, we provide explicit polynomial-time algorithms to solve these two problems for any polynomially perturbed matrix game. For (a), we further provide a functional form for the error-dependent optimal strategy. Last, we translate our results into robust solutions for LPs.

2 - Continuous generalized games
Imre Balog, Miklós Pintér

In our presentation, we examine the existence of equilibrium for finite stochastic games. For this purpose, we introduce a new concept - continuous generalized game - in order to provide a fixed point theorem based proof of the existence for equilibrium of a special class of finite stochastic games (generalized discounted). In our proof, we show that all mentioned stochastic games are so-called continuous generalized game. Regarding continuous generalized games, we show that they have an equilibrium.

3 - Games with partial control
Miklós Pintér

Games where the strategy profiles do not determine the outcome 100% are considered. We present situations where games with partial control emerge naturally. Equilibrium notions for these type of games are introduced. Existence of equilibrium type theorems are also discussed.

■ TB-03

Thursday, 10:00 - 11:30 - Room: C 104

Topics in nonlinear programming

Stream: contributed papers
Invited session
Chair: *Sándor Bozóki*

1 - Convergence of semi-convex functions in CAT(1) spaces and refelction on Douglas Rachford Operator Splitting Algorithm
Hedvig Gal, Miklós Pálfia

The research presents the theory of gradient flows of semi-convex functions established in CAT(1)-spaces, which refers to the a priori estimation of Ohta and Pálfia in CAT(1)-spaces. It denotes that the commutativity property and semi-convexity of the squared distance function is enough to establish the uniqueness, the Evolution Variational Inequalities (EVI) and the contractivity of the gradient flow, similarly as in the CAT(0) setting using the Moreau-Yoshida resolvent. It considers that the discrete and continuous time flows are leading to a Law of Large Numbers in CAT(1)-spaces. The Mosco convergence was previously established for CAT(0) spaces by Kuwae-Shioya and Bačak, and similarly for CAT(1)-space is true that Mosco convergence implies convergence of resolvents, which in turn imply convergence of gradient flows on to the CAT(1) setting. The techniques employ weak convergence in CAT(1)-spaces and cover asymptotic relations of sequences of such spaces introduced by Kuwae-Shioya, including Gromov-Hausdorff limits. The Mosco convergence of non-negative convex lower semi-continuous functions results pointwise convergence of Moreau envelopes in a CAT(0)-space [2]. The related inverse implication was set up by Bačak, Montag and Steidl in CAT(0)-spaces, which extension is represents a new research area on the CAT(1)-spaces. The paper demonstrates a numerical example of Douglas-Rachford operator splitting algorithm, using the Moreau-Yosida envelop theorem.

2 - Different types of feasibility problems via special membership functions
József Dombi

In this talk a universal approach is presented that can be used to find solutions to systems of equations and the feasibility regions described by linear and nonlinear inequalities. We introduce a sigmoid-type and a so-called distending function in order to determine feasibility regions. The combinations of the equalities and inequalities can be expressed by using continuous-valued logical expressions. We also propose new algorithm to find feasible points of linear, nonlinear, convex and non-convex feasibility problems.

3 - Homogeneous convex polyhedra with one unstable equilibrium have at least 7 vertices
Sándor Bozóki, David Papp, Krisztina Regős, Gábor Domokos

The minimal number of vertices of a mono-unstable convex homogenous polyhedron is a challenging open problem. There exists a mono-unstable polyhedra with 18 vertices but it is conjectured non-minimal. Tetrahedra are never mono-unstable. We improve the lower bound from 5 to 7, i.e., the number of vertices of any mono-unstable, convex homogenous polyhedron is at least 7.

Necessary conditions, in the form of multivariate polynomial inequalities, for the existence of mono-unstable convex homogenous polyhedron, are developed. The infeasibility of such systems is proved by semidefinite optimization. The result can be verified independently with the supplemented certificates.

■ TB-04

Thursday, 10:00 - 11:30 - Room: C105

Modeling, Simulation and Optimization

Stream: Large scale optimization and applications
Invited session
Chair: *Kristian Sabo*

1 - Solving sequences of parametrized Lyapunov equations for efficient simulation of parameter influence
Zoran Tomljanovic

We consider a sequence of parametrized Lyapunov equations that can be encountered in many application settings. Repeated solutions of such equations are often intermediate steps of an overall procedure whose main goal is analyzing parameter influence or parameter optimization. We are interested in addressing problems where the parameter dependency of the coefficient matrix is encoded as a low-rank modification to a seed, fixed matrix. We propose two novel numerical procedures that fully exploit such a common structure. The first one builds upon recycling Krylov techniques, and it is well-suited for small dimensional problems as it makes use of dense numerical linear algebra tools. The second algorithm can instead address large-scale problems by relying on state-of-the-art projection techniques based on the extended Krylov subspace. We show the efficiency of the new algorithms on several problems arising in the study of damped vibrational systems and the analyses of output synchronization problems for multi-agent systems.

This is joint work with Davide Palitta, Ivica Nakic and Jens Saak.

2 - A two diffusions stochastic model for epidemic of the SARS-CoV-2 virus
Ivan Papić, Nenad Šuvak, Jasmina Đorđević

A refined version of the classical SEIR (susceptible-exposed-infected-recovered) model for the epidemic of the SARS-CoV-2 virus is proposed. The compartment of infected individuals is divided into four disjoint classes: symptomatic infected individuals (I), superspreaders (P), hospitalized infected individuals (H) and asymptomatic infected individuals (A). The model differentiates the spread of the virus via regular infected individuals and via superspreaders. This assumes two transmission coefficients each representing the spread via (normal) infected individuals and superspreaders. The model is defined through system of stochastic differential equations describing the dynamics of epidemic, where uncertainty in the model is explained through perturbation of transmission coefficients of standard spreaders and superspreaders via two independent Brownian motions with different volatility. The results include proof of existence and uniqueness of the positive global solution of the corresponding system of SDEs as well as the conditions for the extinction of the virus and its persistence in population (persistence in mean). Theoretical results are illustrated via simulations, where the parameters of the model are adjusted based on the data from the early phase of the epidemic in Wuhan (January 4 to March 9, 2020).

3 - Nonlinear distributed estimation in correlated heavy-tailed noise
Manojlo Vukovic, Dusan Jakovetic, Dragana Bajovic, Soummya Kar

We consider distributed inference problems where N agents interconnected in a generic network collaborate to estimate an unknown constant vector parameter while continuously assimilating linear low-dimensional noisy observations of the parameter. Unlike most of existing studies, we focus on the scenario wherein both inter-agent communications and agent sensing are subject to mutually correlated, infinite-variance noises. We present nonlinear distributed estimators that provably work under this challenging setting. Analytical and numerical examples illustrate the findings.

Thursday, 12:00 - 13:30

■ TC-02

Thursday, 12:00 - 13:30 - Room: C 103

Allocations

Stream: Game theory
Invited session
Chair: *Péter Csóka*

1 - An Axiomatization of the Pairwise Netting Proportional Rule in Financial Networks
Péter Csóka, P. Jean-Jacques Herings

We consider financial networks where agents are linked to each other via mutual liabilities. The pairwise netting proportional rule performs one round of pairwise netting between agents that have mutual liabilities and next uses the proportional rule to determine payments. The pairwise netting proportional rule satisfies the basic requirements of claims boundedness, limited liability, priority of creditors, and continuity. It is the only rule to also satisfy the desirable properties of net impartiality, an agent that has two creditors with the same net claims pays the same amount to both creditors on top of pairwise netting, and invariance to mitosis.

2 - TU-games with utility: characterization sets for the u-prenucleolus
Zsófia Dornai, Miklós Pintér

The u-prenucleolus is a generalization of the prenucleolus using utility functions. The u-prenucleolus can also be considered as a generalization of the per-capita prenucleolus. We prove the generalizations of some important theorems about the prenucleolus to the u-prenucleolus, such as Kohlberg's theorem and the theorem of Katsev and Yanovskaya about a sufficient and necessary condition on the unicity of the u-prenucleolus. We also prove a generalization of Huberman's theorem by defining the u-essential coalitions and showing that these coalitions characterize the u-prenucleolus of u-balanced games. Considering the dual of the game, we define the u-anti-prenucleolus, and show that the u-anti-essential coalitions characterize it. Using these results, we get that in the primal game the u-dually-essential coalitions - which generalize the dually essential coalitions defined by Solymosi and Sziklai - characterize the u-prenucleolus. This way, we get a characterization set for the u-prenucleolus that differs from the set of u-essential coalitions.

3 - Nucleolus-type allocations in hierarchies when cooperation is costly
Tamás Solymosi

We consider a multi-agent decision situation when cooperation is possible, but constrained by a hierarchy of the agents. Moreover, having access to a "crucial resource" is necessary for being able to materialize the potential profit-making capabilities of the agents. Making this resource, capable of serving any subgroup of the agents, available has a fixed investment cost. Utilization of the resource is hierarchical, represented by a rooted tree graph. The root represents the resource, each other node of the tree represents one of the agents with a given profit-making potential that can only materialize if all agents on the path to the root are also participating.

We study fair and stable allocations in such "hierarchical joint venture" situations determined by solutions of associated cooperative games. We define the value of a coalition of agents as the sum of the individual potential profits of those members who are connected to the root via other members within the coalition minus the fixed (independent of the coalition to be served) investment cost of the "crucial resource". We consider the standard, the per-capita, and the disruption nucleoli, and investigate whether and how these nucleolus-type allocations can be computed directly from the parameters modeling the "hierarchical joint venture" situation, so there is no need to explicitly generate the exponential-size associated cooperative game and compute its nucleoli with general-purpose algorithms.

■ TC-03

Thursday, 12:00 - 13:30 - Room: C 104

Approximation algorithms for graph problems

Stream: Approximation algorithms
Invited session
Chair: *Gyula Pap*

1 - On the Complexity of Finding Maximum Size Properly Colored Trees and Forests in Edge-Colored Graphs
Gergely Csáji, Yuhang Bai, Kristof Berczi, Tamás Schwarcz

We consider problems related to Properly Colored Spanning Trees. Previous work on properly colored spanning trees has mainly focused on determining the existence of such a tree and therefore the corresponding natural maximization problems were left mostly unexplored. In this paper, we propose an optimization version called Maximum-size Properly Colored Forest problem, where the goal is to find a subset of edges that is both properly colored and forms a forest of the underlying undirected graph. We study the complexity and approximability of the problem based on several different graph classes and different numbers of colors. On the negative side, we provide several inapproximability results which shows that apart from some very special cases, the problem cannot admit a polynomial-time approximation scheme, unless P=NP. On the positive side, we present polynomial-time approximation algorithms as well, for all of our studied settings. Our proof technique relies on the sum of matching matroids defined by the color classes, a connection that might be of independent combinatorial interest. The other maximization problem we consider is the Maximum-size Properly Colored Tree problem, which asks for the maximum size of a properly colored tree. We show that approximating the optimum is significantly more difficult than in the forest case. On the positive side, we show that in the case of complete multigraphs, the problem can be approximated in polynomial-time.

2 - Color-avoiding connected spanning subgraphs with minimum number of edges
Kitti Varga

We call a (not necessarily properly) edge-colored graph edge-color-avoiding connected if after the removal of edges of any single color, the graph remains connected. For vertex-colored graphs, similar definitions of color-avoiding connectivity can be given.

In this talk, we investigate the problem of determining the maximum number of edges that can be removed from either an edge- or a vertex-colored, color-avoiding connected graph so that it remains color-avoiding connected. First, we prove that this problem is NP-hard, and then, we give a polynomial-time approximation algorithm for it. To analyze the approximation factor of this algorithm, we determine the minimum number of edges of color-avoiding connected graphs on a given number of vertices and with a given number of colors. Furthermore, we also consider a generalization of edge-color-avoiding connectivity to matroids.

This is a joint work with József Pintér.

3 - Approximation of disjoint A-paths via fractional matroid matching
Gyula Pap

The disjoint A-path problem, mostly known for Mader's min-max theorems, is related with matroid matching, approximation algorithms, and group theoretical models. We describe a reduction of the fractional disjoint A-path problem to the fractional matroid matching problem - a reduction that also implies an efficient algorithm as the matroid used has a linear representation. There is a 3/2 integrality gap for the matroid matching problem in linear matroids, and also for the disjoint A-paths problem. We explore the relationship between these two gaps, and how they imply approximation algorithms. We would also use this to describe the difficulty of some related open problems, including the problem of finding maximum number of disjoint even A-paths.

■ TC-04

Thursday, 12:00 - 13:30 - Room: C105

P-graph Applications II.

Stream: P-graph algorithms and applications
Invited session
Chair: *Botond Bertok*

1 - Innovations in Public Service Process Management: Enhancing University Enrollment Through P-Graph Methodology
Boglárka Eisinger Balassa, László Buics

This study examines the efficiency of university enrollment processes within public services. We aimed to analyze administrators' workload during enrollment by observing their activities across different workload levels: low, average, and highly overloaded. We meticulously recorded the time spent on enrollment tasks and the frequency of revisiting individual cases. Utilizing the P-graph methodology allowed for precise optimization insights. Our results reveal that administrators often work overtime and face high turnover rates due to being overwhelmed during enrollment periods. We identified significant inefficiencies, with administrators struggling to meet work targets within regular hours. The P-graph methodology provided an unprecedented, detailed view of the process's optimal path and resource allocation, suggesting its application could extend to other public service processes. Our approach also included sitting with administrators to document the exact time spent on tasks and the frequency of revisiting documents, offering a precise evaluation of administrative efficiency and resource needs.

2 - Process Network Solution of a Sport Shooting Event Scheduling Problem
Zsolt Ercsey, Zoltán Kovács, Tamás Storcz

The paper explains the process network solution of amateur sport shooting competition scheduling problems. There is a limited number of firing points where the optimal schedule of the registered competitors should be determined. The competitors are part of teams or sport clubs, where each competitor can be a member of exactly one of the teams. During the competition there are various disciplines with various time limits; where on one hand one competitor may complete one or more disciplines, and on the other hand any discipline can only be completed once by the competitors. Any discipline may be performed by any competitor at any firing point parallelly; moreover, synchronously started and parallelly executed disciplines are grouped into series. This problem is presented as an application of the p-graph methodology. The focus is on the synthesis step, where the corresponding process network is explained and detailed. Based on the maximal structure of the problem a mathematical programming model is generated and solved. Example of a national competition is presented as illustration.

Thursday, 14:45 - 16:15

■ TD-02

Thursday, 14:45 - 16:15 - Room: C 103

Conic and polynomial optimization

Stream: Conic and polynomial optimization
Invited session
Chair: *Miguel Anjos*

1 - Shor convexity, min-max QCQPs and application to min-max regret of nonconvex QPs
Immanuel Bomze, Paula Amaral

Under (finitely many) uncertain scenarios, min-max regret for (possibly nonconvex) Standard QPs can be reduced to a min-max QCQP. En route to narrowing the gap between powerful conic lower bounds and efficient upper bounds, i.e. good feasible values, we will study the apparently novel notion of *Shor convexity* (not to be confused with the well-known notion of Schur convexity) suggested by lifting techniques, and discuss possibilities to use bundle methods for tightening upper bounds. A generalization of the famous Jensen's inequality will be proved as well.

2 - Random Projections for Semidefinite Programming and Polynomial Optimization
Monse Guedes Ayala, Pierre-Louis Poirion, Lars Schewe, Akiko Takeda

Random projections, a dimensionality reduction technique, have been found useful in recent years for reducing the size of optimization problems. In this presentation, we explore using random projections to approximate semidefinite programming (SDP) problems by reducing the size of matrix variables, thereby solving the original problem with less computational effort. We provide some theoretical guarantees on the quality of the projection. We also investigate the performance of the approach on semidefinite relaxations appearing in polynomial optimization, with a focus on combinatorial optimization problems.

3 - Semidefinite liftings for the complex cut polytope
Miguel Anjos, Lennart Sinjorgo, Renata Sotirov

We consider the complex cut polytope: the convex hull of Hermitian rank-one matrices xx*, where the elements of the n-dimensional vector x are complex m-th unit roots. These polytopes find applications in MAX-3-CUT, digital communication, and more generally, complex quadratic programming. For m = 2, the complex cut polytope corresponds to the well-known real cut polytope. We provide an exact description of the complex cut polytope for m = n = 3 and investigate second order semidefinite liftings of the complex cut polytope. For such second order liftings, we show a method for reducing the size of the matrix, without weakening the approximation. We support our theoretical findings with numerical experiments.

■ TD-03

Thursday, 14:45 - 16:15 - Room: C 104

Stochastic optimization and applications I

Stream: Stochastic optimization and applications
Invited session
Chair: *Csaba Fabian*

1 - Improvements of the Q-compression method for constrained stochastic graph traversal problems
Tamás Kegyes, Alex Kummer, Zoltán Süle, János Abonyi

We identified a diverse set of real-life problems belonging to the class of constrained stochastic graph traversal problems. Optimal routing problem of a truck, where the constraints describe the driver's working hours limits and the parking availability belongs to the problem class as well as daily route planning of an electric delivery van, where the constraints are based on the limits of battery capacity and charging options, or disassembly line optimization problem, where a precedence graph describes component removal dependencies and the constraints limit the use of parallel workstations. The common root of these problems is that they lead to a sequential decision problem with a mixed-integer state space. Hence, the classical solution methods are not directly applicable efficiently to the problem class, we turned to reinforcement learning methods. We developed a generally applicable Q-compression method for solving mixed-integer graph optimization problems, which is based on a dynamic discrete representation of the mixed-integer state space and provides a human-interpretable solution to the curse of dimensionality issue. The usability of our method is demonstrated in selected use cases of constrained stochastic graph traversal problems. We are improving the Q-compression method by fine-tuning the framework's discretization method, identifying further problem types for which our method is applicable, and parallelizing the training process for multiple agents.

2 - New algorithms for probability bounds with cherry trees
Edith Kovács

Let A_1 . . . A_n be n arbitrary events in a probability space. The aim is to find upper bounds on the probability of the union events based on calculating only the probabilities of all the intersections of pairs and triplets of events from a dataset. In [1] József Bukszár and András Prékopa proved that with the help of so-called cherry tree graphs, it is possible to give upper bounds on the union of events. They showed that by extending the tree structure, used for the Hunter-Worsley [2, 3] bound, to a cherry tree structure gives an upper bound at least as good as the Hunter-Worsley bound. In the present talk, we exploit the fact that the so-called t-cherry trees used by Bukszár and Prékopa can be regarded also as junction trees. An alternative, equivalent definition of the weight of the cherry tree graph is given, based on which two new algorithms are presented and illustrated on examples.

[1] Bukszár, József, and Andras Prekopa. "Probability bounds with cherry trees." Mathematics of Operations Research 26.1 (2001): 174-192. [2] Hunter, David. "An upper bound for the probability of a union." Journal of Applied Probability 13.3 (1976): 597-603. [3] Worsley, K. J. "An improved Bonferroni inequality and applications." Biometrika 69.2 (1982): 297-302.

3 - Applying random coordinate descent in a probability maximization scheme
Edit Csizmás, Rajmund Drenyovszki, Tamas Szantai, Csaba Fabian

Gradient computation of multivariate distribution functions calls for a considerable effort. A standard procedure is component-wise computation, hence coordinate descent is an attractive choice. This paper deals with constrained convex problems. We apply random coordinate descent in an approximation scheme that is an inexact cutting-plane method from a dual viewpoint. We present convergence proofs and a computational study.

■ TD-04

Thursday, 14:45 - 16:15 - Room: C105

Application of integer programming

Stream: contributed papers
Invited session

1 - Total Earliness-Tardiness problems for Coupled of Tasks scheduling
Gábor Galambos, József Békési, Gyorgy Dosa

We will consider the following problem. Given a Coupled Task Problem (CTP) with n jobs, and exact delay time between two tasks for each job. The objective function is to minimize the total earliness-tardiness (TET) of the schedule in one machine. The practical applications of TET-problems are related to just-in-time production. Both, early and tardy production require special attention, because it is costly if the production performed earlier or later than the expected availability time. Therefore, ideally, optimal schedule occurs when the goods are ready for delivery when they should be available. The structure of the due dates strongly affects both the value of the cost function and the complexity of the problem. We will present two results. First, we consider the special problem, where distinct due dates are given for each job. We prove that this problem is NP-hard, and we present a complexity graph to show the open complexity cases related to the special cases of the problem. In the second problem we will investigate a special case where the delay time and the processing time of the second tasks for each job is equal, and the processing time of the first job is not larger than the constant length of the second tasks. In the considered problem there is a common due date for each job and the due date is "large enough" related to the sum of the sizes of the jobs. For this problem we give a polynomial algorithm to get an optimal solution for the considered problem.

2 - A cost function approximation method for dynamic vehicle routing with LIFO and docking constraints
Markó Horváth, Tamas Kis, Péter Györgyi

In this talk, we consider a dynamic pickup and delivery problem with several practical restrictions, such as last-in-first-out order (LIFO) loading rule and docking constraints. Briefly stated, there is a homogeneous fleet of vehicles to serve pickup-and-delivery requests. Loading orders has to respect the vehicles' capacity limit, while unloading orders has to follow the LIFO rule. The factories have a limited number of docking ports for loading and unloading, which may force the vehicles to wait. The goal is to satisfy all the requests such that a combination of tardiness penalties and traveling costs is minimized. The problem is dynamic in the sense, that the orders are not known in advance, but arrive online, and at certain time points there is an opportunity to make decisions, i.e., to re-plan routes, reflecting on the new information. We propose a cost function approximation method for the problem. At each decision point, we solve the corresponding problem with a perturbed objective function to make solutions flexible for possible future changes. These problems are solved with a variable neighborhood search, for which we apply two existing local search operators, and we also introduce a new one. Our computational experiments show that our solution procedure significantly outperforms the existing methods for the problem on a widely used benchmark dataset.

This work was supported by grant TKP2021-NKTA-01, and by the János Bolyai Research Scholarship.

3 - Analysis of the Workload of Assembly Stations when the Makespan is Minimized in the Presence of Learning Effects
Zakaria Zine El Abidine, Imre Dimény, Tamás Koltai

Efficient assembly lines aim to achieve a shorter makespan to meet customer demands by reducing delivery times. Equally important is the balancing of the workloads of stations to prevent worker overload and to ensure a smooth production flow. Nonetheless, minimizing the makespan and balancing the workload are two conflicting objectives in the presence of learning, as a shorter makespan requires an unbalanced workload distribution. This study introduces a Mixed Integer Linear Programming (MILP) model to analyze the tradeoff between makespan and balanced workload. Using a practical problem, the possible compromises of the two conflicting objectives are explored, and valuable insights for managerial decision-making are provided.

Thursday, 16:45 - 18:15

■ TE-02

Thursday, 16:45 - 18:15 - Room: C 103

Network Optimization

Stream: Discrete Optimization
Invited session
Chair: *Miklós Krész*

1 - Directed k-way Cut and Sparsest Set in Bipartite Graphs
Daniel Szabo, Tamás Király

Given an edge weighted undirected graph G=(V,E), the k-way cut problem asks to find a minimum weight subset C of E such that GC has k connected components. We extend this problem to directed graphs by requiring that GC have k distinct vertices of which none can reach any other. This is a natural extension of the global bicut problem, which admits a less than 2-approximation yet there is a no known hardness result. Very little is known about the directed k-way cut problem, so we show NP hardness by reducing from the hardness of finding the sparsest set in bipartite graphs. This problem asks, given a bipartite graph G=(A,B;E), to find a set of max|A|,|B|+1 vertices with the minimum number of edges between them. We explore hardness of approximation as well as positive results for both of these problems.

2 - Heuristics for finding largest (k,l)-sparse subgraphs
Péter Madarasi, Lóránt Matúz

A multigraph G=(V,E) is (k,l)-sparse if every subset X of the vertices induces at most maxk|X|-l,0 edges. Finding a largest (k,l)-sparse subgraph is a well-studied, polynomial-time solvable problem, which is widely used in rigidity applications and serves as the basis of several combinatorial algorithms. We present a new implementation and compare it with previous solutions on a wide range of random and real-world datasets. The computational study shows that our implementation is consistently faster by one order of magnitude, which we even further improve by new, sophisticated heuristics for fine-tuning the algorithm. We give an in-depth practical analysis of a variety of heuristics for the order in which the algorithm processes the edges. We also implement an algorithm for finding k arc-disjoint r-arborescences in a digraph and k edge-disjoint spanning trees in an undirected graph, which are related to the case l=k. Finally, we give an improved algorithm for the case l=2k when the sparsity condition is required only for the subsets of vertices of size at least 3, which is a necessary condition for 3-dimensional rigidity when k=3.

3 - Approximation algorithm for the weighted connected p-median problem
Miklós Krész, Murat Elhüseyni, Burak Kocuk

In this talk we consider a special case of the well-known p-median problem, the weighted connected p-median problem. The connectivity restriction holds for the placed facilities, i.e. they must form span a connected subgraph. Moreover, we are distinguishing two types of costs: the transportation cost is the cost of the traditional p-median problem between the demands and the facilities, whereas the connection cost is the weight of the minimum spanning tree on the facilities. Nevertheless, for the transportation and the connection we use different cost units. The objective is to place the facilities in such a way to minimize the overall costs (transportation and connection costs).

Our goal is to present a bicrtiteria optimization algorithm for the above problem. In this bictiteria framework we will get an upper bound for the approximation ratio with the assumption that the size of the solution is also bounded with respect to the parameter p. We will also present in which way this theoretical result can initiate practical solution heuristics for the problem.

Acknowledgements: The authors gratefully acknowledge the support of the Slovenian Research and Innovation Agency (ARIS) through grants N1-0223 and N2-0171. Miklós Krész has been also supported by the research program CogniCom (0013103) at the University of Primorska.

■ TE-03

Thursday, 16:45 - 18:15 - Room: C 104

Stochastic optimization and applications II

Stream: Stochastic optimization and applications
Invited session
Chair: *Edit Csizmás*

1 - Plant-wide master production scheduling in the automotive industry: A MILP-approach and a simulation study
Achim Koberstein, Thorben Krüger

Existing approaches for short-term master production scheduling neglect the plant state, plant structure, and lead times; moreover, performance studies regarding their suitability for realistic applications are limited. Therefore, in this paper, we propose a new approach that considers the entire plant structure, its state, and order due dates. We model a fictional but realistic automotive plant using a discrete-event material flow simulation. Through a numerical study, we demonstrate the practical applicability of our approach. Furthermore, we reveal that it can anticipate the behavior of the plant well and can be used to make decisions that minimize due date deviation costs while regulating the shares of components in the plant. We demonstrate that this is particularly important after component blocking, as high additional costs can otherwise arise owing to rework or line stoppages.

2 - Strategic Demand-Side Management: A Probability Maximization-Based Optimization Approach
Rajmund Drenyovszki, Edit Csizmás, Tamas Szantai, Csaba Fabian

The variability introduced by wind and solar energy, along with the increasing number of storage elements, introduces new challenges into the electricity grid. Factors such as weather conditions and individual usage patterns further contribute to uncertainty in energy consumption predictions. Our presentation focuses on a demand-side management model and probability maximization-based optimization approach that schedules controllable devices over various time frames while considering unpredictable energy use. Our solver is designed for straightforward implementation and demonstrates resilience to noise in gradient calculations, offering a practical solution to managing demand amidst the growing adoption of EVs and the fluctuating nature of renewable energy inputs.

■ TE-04

Thursday, 16:45 - 18:15 - Room: C105

OR applications

Stream: contributed papers
Invited session
Chair: *Zoltán Bánhidi*

1 - Complex network approximate symmetries motivated by brain studies
David Hartman, Anna Pidnebesna, Aneta Pokorna, Jaroslav Hlinka

The human brain is a complex system that is very difficult to simulate and analyze in detail. One characteristic of this system is (a)symmetry in its structure. It is even assumed that the mentioned (a)symmetry influences various functional properties of the brain. One of the modern methods for analyzing this system is represented by complex networks. These networks describe both the structural and functional connectivity of the brain and many characteristics of the system, including potential symmetry, can be inferred from their structure. Recently, it has been shown that non-trivial symmetries based on graph automorphisms exist in many complex networks. However, such symmetries do not account for uncertainty in the edges. Therefore, a relaxed alternative allowing approximate automorphisms has recently been proposed. However, the proposed method has some shortcomings. Therefore, in this paper, we propose an alternative approach using a recently proposed optimization method from the field of graph matching with a modification for variable inclusion of fixed points. In addition to the proposed method itself, we also propose a method for testing similar algorithms for approximate symmetries on suitably constructed random models motivated by, among other things, networks in the brain.

2 - Quantifying the impact of outlier management techniques on digital country rankings
Zoltán Bánhidi, Imre Dobos

The objective of our study is to create rankings of European Union (EU) member states based on objective weights that provide a comprehensive overview of their digital and economic development. We also aim to examine the impact of outlier management techniques, such as winsorising, on these rankings. To accomplish this, we utilised a macro-level cross-sectional dataset that comprises the principal dimensions of the Digital Economy and Society Index, as published by the European Commission, along with the GDP per capita and AIC indicators from economic statistics. In one version of the dataset, extreme values in the raw GDP per capita data were treated with winsorising, while in another version, they were left untreated. The efficiency indicators were used to rank EU Member States based on the synthesis of the digital and economic dimensions using decision-theoretic methods, two DEA models, and a TOPSIS model. The rankings aim to characterise the digital-economic strengths of EU countries and the digital divide found within the EU, as well as evaluate the impact of outlier management. The rankings are also compared with those of the original DESI scoring model to test the contribution of macroeconomic indicators to the results.

Friday, 9:00 - 10:00

■ FA-05

Friday, 9:00 - 10:00 - Room: E III

Plenary III

Stream: Plenaries
Invited session
Chair: *Marianna E.-Nagy*

1 - Performance estimation of optimization methods: a guided tour
François Glineur

Identifying the worst-case behaviour of an optimization method is a question that can itself be cast as an optimization problem. This is the main idea behind performance estimation, initially proposed by Drori and Teboulle in 2014. In this framework, one seeks to compute the exact convergence rate of a given black-box optimization algorithm over a given class of problem instances. In many cases, this computation can be reformulated into a finite-dimensional, tractable semidefinite program. Solving this program provides an exact, unimprovable convergence rate, a mathematical proof guaranteeing this rate and a problem instance where this worst-case behaviour is achieved. This is useful to compare efficiency across methods, tune algorithmic parameters (such as step-size) and, ultimately, design new methods with improved behaviour.

In this talk we will survey a few of the main achievements in the area of performance estimation, including the exact convergence rates of the gradient method over the full range of constant step-sizes, both on smooth (strongly) convex and smooth nonconvex functions, how to deal with methods involving linear operators and the exact convergence rate of the last iterate in subgradient methods, as well as some open questions related to performance estimation.

This talk will present joint work with Nizar Bousselmi, Julien Hendrickx, Panagiotis Patrinos, Teodor Rotaru and Moslem Zamani.

Friday, 10:15 - 11:45

■ FB-02

Friday, 10:15 - 11:45 - Room: C 103

Decision theory

Stream: Decision theory
Invited session
Chair: *Sándor Bozóki*

1 - Estimating army effectiveness using pairwise comparison matrices
Kristóf Ábele-Nagy

Pairwise comparison matrices are widely used in multi-criteria decision making, ranking, and other problems. In the presented application, the relative efficiency of the armies of participating countries or other parties in a war is analyzed. The ratio of each country's losses caused to each other can be written in a pairwise comparison matrix. Since not every country meets in battle with every other country involved, we get an incomplete pairwise comparison matrix due to missing proportions. From this matrix, taking its weight vector, the relative efficiency indicators for the armies of each country can be obtained, with which the parties that are not at war with each other can also be compared. By recalculating the matrix at different times of the war, we can also get an idea of the evolution of the relative efficiencies over time. The method is demonstrated on randomly generated data, and also on historical data at the very beginning of the First World War, which is the ultimate goal of application.

2 - Allocations based on pairwise comparisons
Zsombor Szádoczki, *Sándor Bozóki*

The allocation of financial or other scarce resources is a common optimization problem connected to group decision making. This paper aims to suggest a reasonable approach to these issues based on pairwise comparisons. In the proposed model, each decision maker provides the pairwise comparisons (or the direct evaluation if possible) of a subset of the alternatives. The union of all comparisons can result in a complete, an incomplete or a multigraph, i.e., some of the elements have never been compared, while others have been compared several times. If there are multiple comparisons between two alternatives, then we use their geometric mean in the evaluations. Finally, the weight calculation techniques connected to incomplete pairwise comparison matrices are used to determine the allocated ratio of each alternative.

3 - Aggregation of pairwise comparison matrices: A clustering approach
Kolos Ágoston, *Sándor Bozóki*, *Laszlo Csato*

We consider clustering in group decision making where the opinions are given by pairwise comparison matrices. In particular, the k-medoids model is suggested to classify the matrices as it has a linear programming problem formulation. Its objective function depends on the measure of dissimilarity between the matrices but not on the weights derived from them. With one cluster, our methodology provides an alternative to the conventional aggregation procedures. It can also be used to quantify the reliability of the aggregation. The proposed theoretical framework is applied to a large-scale experimental dataset, on which it is able to automatically detect some mistakes made by the decision-makers.

■ FB-03

Friday, 10:15 - 11:45 - Room: C 104

Approximation algorithms for scheduling problems

Stream: Approximation algorithms
Invited session
Chair: *Tamas Kis*

1 - Analysis of an Approximation Algorithm for Coupled Task Scheduling with Equal Lengths of Tasks for Minimizing the Sum of Completion Times
József Békési, *Gyorgy Dosa*, *Gábor Galambos*

In the considered coupled task problem (CTP) we have to schedule n jobs on a single machine, each of them consisting of two tasks with exact time delays in between, while the objective is to minimize the total of job completion times. We analyze a greedy type algorithm – called A – from worst case point of view, and we give bounds for the asymptotic behavior of A for the special case where each task has equal length p. For this case, the best-known upper bound on the asymptotic performance ratio of algorithm A has been smaller or equal than 5/3. We improve this bound to 1.5426... and we give a closed formula for the upper bound for every p >= 1. We use constructions to calculate lower bounds and thus give narrow intervals for the asymptotic behavior of the algorithm A as a function of the parameter p. The bounds are tight for p=1 and p=2, the values are 1.18350341... and 1.348612... respectively.

2 - Improving bounds on approximation algorithms for the Triangle Scheduling problem using Mixed Integer Quadratic Programming
Nóra Büki, *János Balogh*, *József Békési*, *Gyorgy Dosa*, *Zsolt Tuza*

The Triangle Scheduling problem is a model for non-preemptive mixed criticality scheduling of unit length jobs. Two algorithms, Greedy and Bintree are both known to alpha-approximate the optimum value (with alpha = 1.5 and alpha = 2ln(2), respectively), but the tightness of those upper bounds is still an open question, with a best known gap as large as 0.45 for Greedy. We provide Mixed Integer Quadratic Programming formulations for both algorithms that, for certain small n, give an n-length input whose ALG makespan is the largest it can be for that input length. We are then able to use these small inputs to generate increasingly larger ones that provide better lower bounds, significantly reducing the gaps between the lower and upper bounds (LB=1.27 for Greedy and LB=1.35 for Bintree), and gaining insight about the different structures of inputs each algorithm performs badly on.

3 - Joint replenishment meets scheduling
Tamas Kis, Péter Györgyi, Timea Tamasi

In the talk I give an overview on the joint replenishment problem combined with machine scheduling. In the joint replenishment problem a set of requests must be satisfied by ordering the corresponding items. Each request has a release date, and specifies an item type. Ordering an item incurs a fixed cost, which depends only on the item type. Requests for the same item can be fulfilled together to save ordering costs, but there is a penalty for late fulfillment. There are several variants of this classic O.R. problem, but its combination with machine scheduling has only recently begun to be investigated. In the combined problem, after ordering some items, they have to be processed on a machine to fulfill the requests. I will summarize complexity results, and describe approximation as well as online algorithms for different cost functions under various conditions.

■ FB-04

Friday, 10:15 - 11:45 - Room: C105

Methods of optimization

Stream: contributed papers
Invited session
Chair: *Mátyás Koniorczyk*

1 - Graph cliques and quantum annealing
Mátyás Koniorczyk, Kristóf Váradi, Sandor Szabo

We explore the usability of quantum annealers on graph clique problems of Erdős-Rényi graphs. These random graphs offer a way to generate well-parametrized sets of benchmark problems with known interesting structural properties. We calculate clique numbers using actual quantum hardware as well as with Cliquer for comparison; the latter is amongst the most efficient algorithms for the problem. We compare the timings and the quality of the solution, and analyze the effect of the quantum annealing parameters on the solution. We find that in certain cases the quantum hardware can be competitive.

2 - New interval-based training technique to parameter robustness
Attila Szász, Balázs Bánhelyi

Today's artificial neural networks appear in many scientific fields and have a wide range of applications, for example, they are widely used for image and speech recognition. Over the years, the accuracy of the networks has continuously improved, but many studies have shown that these networks are also not error-free. Many technologies have been developed to reduce the probability of error, but most of them look for examples of adversities in the input space and try to make neural networks more robust by using them. There is a much less technique to search for aversality in the smaller distances of the weight matrices of the neural network. However, this type of adversity is magnified in that area, where taught neural networks are evaluated with much lower accuracy. In these quantized neural networks, the results differ greatly from the expected result.

In our presentation, we present this problem and compare the robustness in the input space and the parameter space of the network. The effectiveness of methods based on floating-point and interval evaluations was shown. A new training method was implemented to parameter robustness and the results show that it is more effective than previous techniques.

3 - Particle Filter Optimisation algorithms for robust optimisation
Éva Kenyeres, Alex Kummer, János Abonyi

Population-based optimization algorithms gained high interest in the last few decades as they reach outstanding performance by propagating not only one but many candidate solutions in the search space. The Particle Filter Optimisation (PFO) concept is one novel representative of these techniques, which is based on the probabilistic approach of the Particle Filter state estimation algorithm. The PFO algorithm moves weighted sample elements in the search space thus a probability distribution of the elements is evolved. This carries information about the shape of the objective function. The related literature contains the description of several variants of the PFO aiming to find the global optimum. However, the opportunity to use it for robust optimization has not been investigated yet. As in practical problems besides uncertain factors often not the global but a more stable local optimum with high performance gives the most desirable solution, our research deals with the possible robust optimization applications of the algorithm. Our presentation will address the following points: 1) Introduction of the PFO algorithm and its crucial tunable elements. 2) Some ideas on how to use PFO for robust optimization, e.g., combined with clustering. 3) Experiment results on a benchmark function and on a practical problem from the chemical engineering field. Results verify, that the PFO algorithm holds a promising potential for robust optimization aims due to its probabilistic nature.

Friday, 12:15 - 13:15

■ FC-05

Friday, 12:15 - 13:15 - Room: E III

Plenary IV

Stream: Plenaries
Invited session
Chair: *Giancarlo Bigi*

1 - Decision rules for sequential decision-making under uncertainty
Merve Bodur

Sequential decision-making emerges in a broad range of fields and is often impacted by uncertainty. Multistage stochastic programming (MSP) and multistage adjustable robust optimization (MSARO) are suitable modelling frameworks for sequential decision-making under uncertainty, among others. Those problems are theoretically and computationally challenging and, as such usually solved by means of approximations. In that regard, a commonly employed approach is decision rules (DRs), which restrict the policies to follow a certain functional form of the observed random outcomes. In this talk, we will review traditional as well as recently proposed primal and dual DRs for general MSP and MSARO problems. Our review will include two-stage DRs and Lagrangian dual DRs, which make solution algorithms designed for two-stage problems amenable to generating high-quality policies for multistage problems. In particular, for MSPs with potentially mixed-integer recourse, our review will also feature a Markov-chain-based variant of two-stage DRs to leverage the underlying stochastic process and a certain class of mixed-integer DRs for nonlinear MSPs with a large number of stages. The latter, by design, leads to smooth restricted problems (making stochastic gradient decent methods amenable) and highly interpretable decision policies. We will present numerical results on applications from various domains to illustrate the presented ideas.

Friday, 13:15 - 13:30

■ FD-05

Friday, 13:15 - 13:30 - Room: E III

Closing

Stream: Plenaries
Invited session
Chair: *Tibor Illés*

Streams

Advances in theory and practice of interior-point methods

Zsolt Darvay
Babes-Bolyai University
zsolt.darvay@ubbcluj.ro

Goran Lesaja
Georgia Southern University
goran@georgiasouthern.edu

Track(s): 3

Approximation algorithms

Tamás Király
Eötvös Loránd University
tamas.kiraly@ttk.elte.hu

Track(s): 3

Conic and polynomial optimization

Miguel Anjos
University of Edinburgh
anjos@stanfordalumni.org

Track(s): 2

contributed papers

Track(s): 3 4

Decision theory

Sándor Bozóki
Institute for Computer Science and Control, Hungarian Academy of Sciences (MTA SZTAKI)
bozoki.sandor@sztaki.mta.hu

Sándor Bozóki
SZTAKI
bozoki.sandor@sztaki.mta.hu

Track(s): 2

Discrete Optimization

Laszlo Vegh
LSE
l.vegh@lse.ac.uk

Track(s): 2 3

Game theory

Michel Grabisch
Universite Paris I - Pantheon-Sorbonne
michel.grabisch@univ-paris1.fr

Miklós Pintér
Corvinus University of Budapest
pmiklos@protonmail.com

Tamás Solymosi
Corvinus University of Budapest
tamas.solymosi@uni-corvinus.hu

Track(s): 2

Large scale optimization and applications

Natasa Krejic
University of Novi Sad Faculty of Science
natasak@uns.ac.rs

Kristian Sabo
University Osijek
ksabo@mathos.hr

Track(s): 4

P-graph algorithms and applications

Botond Bertok
Széchenyi István University
botondbertok@gmail.com

Zoltán Süle
University of Pannonia
sule@dcs.uni-pannon.hu

Track(s): 4

Plenaries

Giancarlo Bigi
Universita' di Pisa
giancarlo.bigi@unipi.it

Track(s): 1 5

Stochastic optimization and applications

Edit Csizmás
John von Neumann University
csizmas.edit@nje.hu

Csaba Fabian
John von Neumann University
fabian.csaba@gamf.uni-neumann.hu

Track(s): 3

Session Chair Index

Author Index

Session Index

Wednesday, 8:30 - 9:00

Wednesday, 9:00 - 10:00

Wednesday, 10:00 - 11:30

Wednesday, 12:00 - 13:30

Wednesday, 14:45 - 16:15

Wednesday, 16:45 - 18:15

Thursday, 9:00 - 10:00

Thursday, 10:00 - 11:30

Thursday, 12:00 - 13:30

Thursday, 14:45 - 16:15

Thursday, 16:45 - 18:15

Friday, 9:00 - 10:00

Friday, 10:15 - 11:45

Friday, 12:15 - 13:15

Friday, 13:15 - 13:30