Challenge EURO/ROADEF 2022

Truck loading

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Contents

1 Introduction

RENAULT's supply chain spans over 40 plants in 17 countries and 1500 suppliers. Every week, 6000 trucks deliver parts from suppliers to plants. The filling rate of these trucks is critical, since the inbound transportation annual budget is over several hundred millions of euros.

The objective is to pack a set of items from suppliers into stacks and to pack the stacks into trucks which deliver the plants, in order to minimize (a) the number of the trucks used and (b) the inventory in the plants due to early deliveries. Items have a time window [earliest arrival time, latest arrival time] for their delivery that ranges from 1 to 5 days. It is possible to deliver the items early (for instance at their earliest arrival time) in order to better fill the trucks, but the early deliveries generate inventory costs for plants. Inventory costs are very sensitive at Renault, since they also represent several hundred of millions of euros. So the best solution is to deliver items to plants at the latest possible time, while minimizing the number of trucks used.
Renault hired trucks from transporters to deliver the items from the suppliers to the plants. These hired trucks are defined in an annual planning and are called "planned" trucks. All items must be delivered in time to their plants. For every item, there is at least one planned truck with an arrival time included in the item's time window, that is the planned truck can theoretically deliver the item to the plant in time. But it may occur that there are not enough planned trucks to deliver all the items in time. In such cases, "extra" trucks can be called from the transporter to deliver the items. These extra trucks cost more than planned trucks.

In terms of data volume, a large instance can contain up to 260000 items and 5000 planned trucks, over a horizon of 7 weeks.

2 Problem description

Three-dimensional trucks contain stacks packed on the two-dimensional floor. Each stack contains items that are packed one above another. There are three correlated dimensions (length, width, height) and an additional weight dimension. We will call horizontal dimensions the length and the width. The position of any element (item or stack) is described in a solution by its position in the three axis X, Y and Z. There is one origin per truck, corresponding with the leftmost downward position (cf Figure 1).

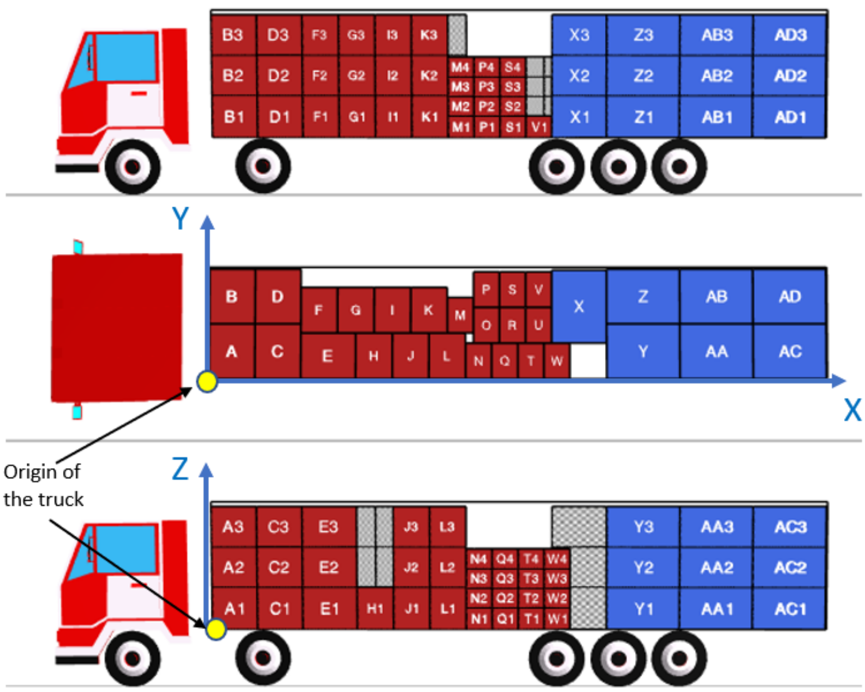

Figure 1: Views from top to bottom : right view, above view and left view of a truck, seen from the rear of the truck. The truck contains 96 items (A1 to AD3) packed into 30 stacks (A to AD).

2.1 Items

An item i is a packaging containing a product IR_i. It is characterized by its space dimensions length/width/height (IL_i, IW_i, IH_i), its weigth IM_i, a stackability code IS_i. By convention, the length is larger than the width.

The stackability code is used to define which items can be packed into the same stack: only items with the same stackability code can be packed into the same stack. Items with the same stackability code share the same length and width. But the opposite is false : items with identical length/width may not share the same stackability code.

An item i is associated with maximal stackability ISM_i. It represents the maximal number of items i in a stack.

The items may be rotated only in one dimension (horizontal), i.e. the bottom of an item remains the bottom. An item may be oriented lengthwise (the item's length is parallel with the length of the truck), or widthwise (the item's length is parallel with the width of the truck). In Figure 1, the stack X is oriented widthwise, while the stack Y is oriented lengthwise. Item i may have a forced orientation IO_i, in which case the stack s containing item i must have the same orientation.

An item i must be delivered to a plant IP_i in a time window $[IDE_i, IDL_i]$, with IDE_i is the earliest arrival time and IDL_i the latest arrival time. An item i is produced by a supplier IU_i. Parameters IP_i, IU_i, IR_i and $[IDE_i, IDL_i]$ will determine which trucks can be used to load the item i.

An item i is loaded at a supplier dock IK_i and unloaded at a plant dock IG_i. These docks will impact the placement of items into trucks.

An item i has an unitary inventory cost IC_i, which will be used to calculate the inventory cost in the objective function.

An item i has a nesting height $\widehat{IH}_i$, which is used for the calculation of the height of stacks : if item i_1 is packed above item i_2, then the total height is $IH_{i_1} + IH_{i_2}$ - $\widehat{IH}_{i_1}$ (cf Figure 2). For the majority of items, the nesting height is equal to zero.

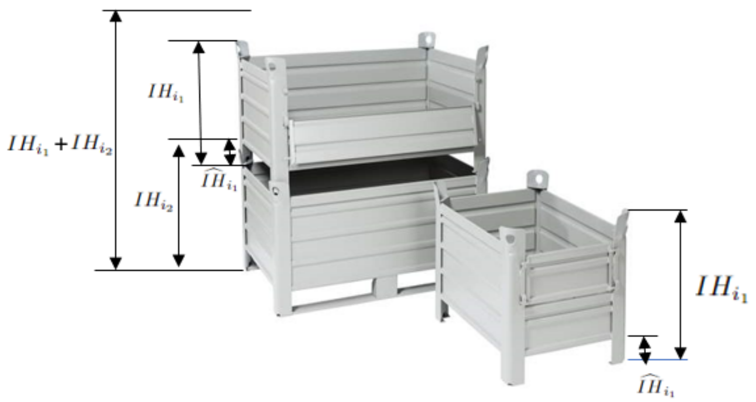

Figure 2: Example of a stack of 2 items with nesting height

2.2 Trucks

A planned truck t is characterized by its dimensions length/width/height (TL_t, TW_t, TH_t), its maximum authorized loading weight TM_t^m, its cost TC_t. The truck's cost represents a fixed cost that does not depend on the number of items loaded into the truck.

A truck t is planned to arrive at the plant TP_t at the arrival time TDA_t. It can pickup the set of products $\widetilde{TR}_t$ from a set of suppliers $\widetilde{TU}_t$. We define the set of products related to a truck because it happens that a truck cannot pickup all the products of a supplier. It occurs when a product must be unloaded at a specific plant dock and the truck does not stop at this plant dock. There is a loading order between the picked-up suppliers.

As said in the introduction, in case there are not enough planned trucks to deliver all the items in time, extra trucks can be called from the transporter to deliver the items. An extra truck $\widehat{t}_j$ is based on a planned truck, i.e. $\widehat{t}_j$ has the same characteristics than a planned truck t ($TL_{\widehat{t}_j} = TL_t$, $TW_{\widehat{t}_j} = TW_t$, $TH_{\widehat{t}_j} = TH_t$, $TM_{\widehat{t}_j}^m =$

TM_t^m, $TDA_{\widehat{t_j}} = TDA_t$, $\widetilde{TR}_{\widehat{t_j}} = \widetilde{TR}_t$, $\widetilde{TU}_{\widehat{t_j}} = \widetilde{TU}_t$), except for the cost ($TC_{\widehat{t_j}} = (1+\alpha^E)TC_t$, with the cost coefficient α^E for extra trucks). We can call several identical extra trucks $\widehat{t_j}$ if needed. There is no constraint on the number of extra trucks. Of course, it does not make sense to call more extra trucks than the number of items.

An example to illustrate the use of extra trucks : 200 items of supplier A must arrive at the latest at date D. There is only one planned truck t to deliver the items on time, and a truck can load a maximum of 80 items. So 80 of the 200 items will be delivered by the planned truck t. And two extra trucks $\widehat{t}_1$ and $\widehat{t}_2$ (with the same characteristics than t) will be called to deliver the remaining 120 items.

All the stacks packed into truck t must satisfy a maximal density TEM_t.

A truck t is associated with a flag "stack with multiple docks" TF_t (yes/no). If (TF_t = yes), a stack packed into truck t may include items associated with different plant docks (a maximum of 2 plant docks, with consecutive loading order). If (TF_t = no), all the stacks packed into truck t must have a single plant dock among its items.

There is a maximal total weight $TMM_t\gamma$ for all the items packed above the bottom item associated with product γ in any stack loaded into truck t.

2.3 Stacks

While items and planned trucks are parameters for the problem, stacks represent the results of the optimization. A stack contains items, packed one above the other, and is loaded into a truck. A stack s is characterized by its dimensions length/width/height (sl_s, sw_s, sh_s) and its weight sm_s. The length and width of a stack are the length and width of its items (all the items of a stack share the same length/width). A stack's orientation is the orientation of all its items. There is a unique orientation (lengthwise or widthwise) for all the items of a stack. The stack is characterized by the coordinates of its origin (sx_s^o, sy_s^o, sz_s^o) and extremity points (sx_s^e, sy_s^e, sz_s^e).

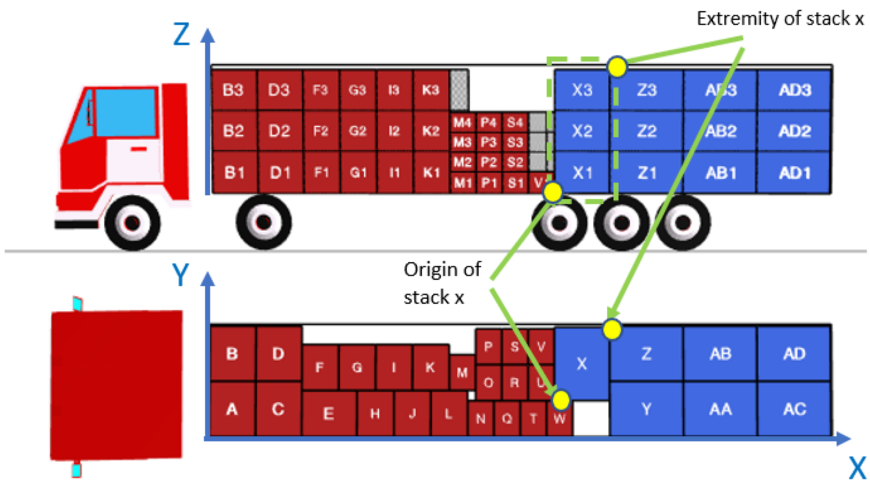

Figure 3: Origin and extremity points of a stack

As shown in Figure 3, the origin is the leftmost bottom point of the stack (the point nearest the origin of the truck), while the extremity point is the rightmost top point of the stack (the farthest point from the origin of the truck). The coordinates of origin and extremity points of a stack must satisfy the following assumptions :

- $sx_s^e - sx_s^o = sl_s$ and $sy_s^e - sy_s^o = sw_s$ if stack s is oriented lengthwise
- $sx_s^e - sx_s^o = sw_s$ and $sy_s^e - sy_s^o = sl_s$ if stack s is oriented widthwise
- $sz_s^o = 0$ and $sz_s^e = sh_s$

A stack's weight sm_s is the sum of the weights of the items it contains : $sm_s = \sum_{i \in I_s} IM_i$.
The stack's height sh_s is the sum of the heights of its items, minus their nesting heights (cf Figure 2) : $sh_s = \sum_{i \in I_s} IH_i - \sum_{i \in I_s \text{ and } i \neq bottom\_item} \widehat{IH}_i$.

2.4 Objective

The objective is to minimize simultaneously the total transportation and inventory cost. The transportation cost is the total cost of planned and extra trucks, it is a fixed cost which depends only on the number of trucks, not the number of loaded items. The inventory cost is a variable cost due to early deliveries of items, calculated on the interval between the arrival time of items at the plant and the latest arrival time of these items. The inventory cost does not include a fixed part.

2.5 Constraints

There are 4 sets of constraints :

- item constraints that ensure the compatibility between items and trucks, that is which items can be loaded into which trucks.
- stack constraints that control the composition of stacks, mainly which items can be packed into the same stack
- placement constraints related to the placement of stacks into trucks, especially the placement order of stacks according to suppliers' and plant's docks
- weight constraints that impose the maximal authorized weights of stacks in the truck, and also specifically on each of the 2 axles of the truck.

3 Problem model

3.1 Notations

By convention, parameters are declared with uppercase letters, variables with lowercase letters and sets with widetile uppercase letters (ex: $\widetilde{U}$). Parameters related to items are prefixed with I, parameters related to trucks with T and variables related to stacks with s.
Parameters and variables related to weight constraints will be defined in Section 3.3.4.

3.1.1 Parameters

The parameters are defined in input files of each data instance.

$\widetilde{I}$: set of items i
$\widetilde{P}$: set of plants p
$\widetilde{U}$: set of suppliers u
$\widetilde{G}$: set of plant docks g
$\widetilde{K}$: set of supplier docks k
$\widetilde{\gamma}$: set of products γ

IL_i, IW_i, IH_i, $\widehat{IH}_i$, IM_i : length/width/height/nesting height/weight of item i
IS_i : stackability code of item i
ISM_i : maximal stackability of item i
IR_i : product ($\in \widetilde{\gamma}$) of item i
IO_i : forced orientation of item i
IDE_i, IDL_i : earliest and latest arrival time of item i
IP_i : destination plant of item i
IG_i : plant dock of item i
IU_i : supplier of item i
IK_i : supplier dock of item i
IC_i : inventory cost of item i

$\widetilde{TR}_t$: set of candidate products picked-up by truck t
$\widetilde{TU}_t$: set of candidate suppliers picked-up by truck t
$\widetilde{TK}_{ut}$: set of candidate supplier docks k of supplier u loaded into truck t
$\widetilde{TG}_{pt}$: set of candidate plant docks g of plant p delivered by truck t

TL_t, TW_t, TH_t, TM_t^m : length/width/height/max authorized loading weight of truck t
TP_t : destination plant of truck t
TDA_t : arrival time of truck t at TP_t
$TMM_t\gamma$: maximal total weight of all the items packed above the bottom item associated with product γ in any stack of truck t
TF_t : flag "stack with multiple docks" for truck t
TEM_t : maximal density of stacks in truck t

TC_t : cost of truck t
TE_t : supplier loading order for truck t : it is a list indexed by the elements of $\widetilde{TU}_t$ containing for each supplier its loading order
TKE_{ut} : dock loading order of supplier u for truck t : it is a list indexed by the elements of $\widetilde{TK_{ut}}$ containing for each supplier dock its loading order
TGE_{pt} : dock loading order of plant p for truck t : it is a list indexed by the elements of $\widetilde{TG_{pt}}$ containing for each plant dock its loading order

The dock loading order TKE_{ut} ranges from 1 to $|\widetilde{TK_{ut}}|$. When the loading order a supplier dock k is 1, it means that items associated with supplier dock k should be loaded first in the truck t, i.e. these items should be placed at the front of the truck, near the driver's cabin.
The dock loading order TGE_{pt} ranges from 1 to $|\widetilde{G_{pt}}|$. When the loading order of a plant dock g is 1, it means that items associated with plant dock g should be loaded first in the truck t, i.e. these items should be placed at the front of the truck, near the driver's cabin.

α^T : coefficient of transportation cost in the objective function
α^I : coefficient of inventory cost in the objective function
α^E : coefficient of cost for extra trucks

3.1.2 Variables

The variables describe the solution and must be written in results files of each data instance.

$\widetilde{T}$: set of used trucks t (planned and extra trucks)
$\widetilde{TS}_t$: set of the stacks packed into truck t

$\widetilde{SI}_s$: set of the items of stack s
$\widetilde{SG}_s$: set of the plant docks of stack s
sl_s, sw_s, sh_s, sm_s : length, width, height, weight of stack s
sx_s^o, sy_s^o, sz_s^o : coordinates of the origin point of stack s
sx_s^e, sy_s^e, sz_s^e : coordinates of the extremity point of stack s
so_s : orientation of stack s
su_s : supplier of stack s
sk_s : supplier dock of stack s
st_s : truck of stack s

ida_i : arrival time of item i

3.2 Objective

The objective function is : Min $(\alpha^T \times (\sum_{t\in\widetilde{T}} TC_t) + \alpha^I \times (\sum_{i\in\widetilde{I}} IC_i * (IDL_i - ida_i)))$

The difference (IDL_i - ida_i) is computed in terms of days. Consequently, if an item is delivered to the plant at the same day than its latest arrival time, then the inventory cost for this item is zero.

3.3 Constraints

3.3.1 Items constraints

- All items must be packed into stacks, which must be loaded into trucks (planned or extra) **(I1)**

- An item i can be loaded into a truck t if and only if :
  - t arrives at the item's plant IP_i **(I2)**
  - t can pickup the item's product IR_i **(I3)**
  - t stops by the item's supplier IU_i **(I4)**
  - t arrives at the plant in the item's time window : $IDE_i \leq TDA_t \leq IDL_i$ **(I5)**

3.3.2 Stacks constraints

- All the items packed in a stack must share the same supplier, plant, stackability code and supplier dock. **(S1)**

- For any stack s packed into truck t and if (TF_t = no), then all the items of stack s must share the same plant dock. **(S2)**

- For any stack s packed into truck t, if (TF_t = yes), s may contain items with 2 plant docks with consecutive loading orders. **(S3)**
  There is a restriction : for every stackability code SC present among the stacks of a truck t, only one stack with the stackability code SC can contain items with 2 plant docks.

- If one item i of a stack s has a forced orientation IO_i, then all the items of stack s must share the same orientation ($so_s = IO_i$). **(S4)**
  Consequently, there cannot be 2 different forced orientations of items in the same stack.

- For any stack s packed into truck t, the total weight of the items packed above the bottom item associated with product γ must not exceed the maximal weight $TMM_t\gamma$: $\sum_{i \in \widetilde{SI}_s \text{ and } i \neq bottom\_item} IM_i \leq TMM_t\gamma$ **(S5)**

- The number of items packed into a stack s must not exceed the smallest "max stackability" ISM_i of the items i present in stack s. **(S6)**
  In other words, for each item i of a stack s, the number of items of s must not exceed ISM_i.

- The density of a stack s must not exceed the maximal stack density defined for the truck t into which the stack s is loaded : $\frac{sm_s}{sl_s \times sw_s} \leq TEM_t$ **(S7)**

3.3.3 Placement constraints

- The placement of a stack s into a truck t must not exceed the truck's dimensions :
  $\forall t \in \widetilde{T}, \forall s \in \widetilde{TS}_t, \quad sx^e_s \leq TL_t$ and $sy^e_s \leq TW_t$ and $sz^e_s \leq TH_t$ **(P1)**

- The stacks packed into a truck t cannot overlap :
  $\forall t \in \widetilde{T}, \forall s_1, s_2 \in \widetilde{TS}_t$ with $sx^o_{s_1} \leq sx^o_{s_2}$, if $(sx^o_{s_2} < sx^e_{s_1})$ then $sy^o_{s_2} \geq sy^e_{s_1}$ or $sy^e_{s_2} \leq sy^o_{s_1}$ **(P2)**
  If s_1 and s_2 overlap in the X axis, then they cannot overlap in the Y axis.

- Any stack must be adjacent to another stack on its left on the X axis, or if there is a single stack in the truck, the unique stack must be placed at the front of the truck (adjacent to the truck driver) :

  $\forall t \in \widetilde{T}, \forall s_1 \in \widetilde{TS}_t$ with $sx^o_{s_1} > 0$, $\exists s_2 \in \widetilde{TS}_t$ with $(sx^e_{s_2} = sx^o_{s_1})$ and $(sy^o_{s_2} \in [sy^o_{s_1}, sy^e_{s_1}]$ or $sy^e_{s_2} \in [sy^o_{s_1}, sy^e_{s_1}])$ **(P3)**

  In Figure 4, the placement of stacks A and B is forbidden (and by consequence the placement of stack D is also forbidden since it is adjacent only to the forbidden stack B), while the placement of all the other stacks is allowed. The business motivation is that in case the driver should brake abruptly, any stack should be retained either by another stack or by the front of the truck.

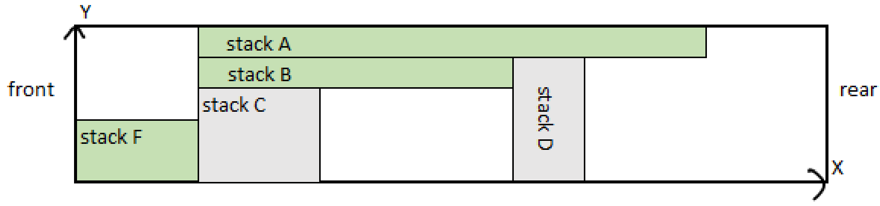

Figure 4: The placements of stacks A and B are forbidden

- Stacks must be placed in an increasing fashion from the front to the rear of the truck, (1) according to the suppliers' pickup order, and among the stacks of the same supplier, stacks must be placed in an increasing fashion (2) according to the supplier dock loading order, and among the stacks with the same supplier and supplier dock, stacks must be placed in an increasing fashion (3) according to the plant dock loading order. We do not try to optimize the unloading of the stacks, other than the satisfaction of the 3 precited orders. **(P4)**

  $\forall t \in \widetilde{T}, \forall s_1 \in \widetilde{TS}_t, \forall s_2 \in \widetilde{TS}_t,$
  (1) if $TE_{u_{s_1}} < TE_{u_{s_2}}$ then $sx^o_{s1} \leq sx^o_{s2}$
  (2) if $(u_{s_1} = u_{s_2}$ and $KE_{k_{s_1}} < KE_{k_{s_2}})$ then $sx^o_{s1} \leq sx^o_{s2}$
  (3) if $(u_{s_1} = u_{s_2}$ and $KE_{k_{s_1}} = KE_{k_{s_2}}$ and $GE_{g_{s_1}} < GE_{g_{s_2}}, \forall g_{s_1} \in \widetilde{SG_{s_1}}, \forall g_{s_2} \in \widetilde{SG_{s_2}})$ then $sx^o_{s1} \leq sx^o_{s2}$

  In annexes, Figures 5 to 9 illustrate this constraint. Figure 7 illustrates the specific case of stacks with 2 plant docks. These stacks must satisfy the constraint for each of their 2 plant docks.

  Supplier docks may be unknown for some trucks, in which case there is no constraint on the loading order of supplier docks in these trucks.
  Plant dock loading order may be equal to 0 for several plant docks in some trucks, in which case, there is no constraint on the loading order for these plant docks in these trucks.

3.3.4 Weight constraints

A truck is made up of a tractor (which includes the driver cabin) and a trailer (where the stacks are loaded), as shown in Figures 10 and 11. Parameters will be defined either for the tractor or the trailer, while variables are defined only for the trailer.

3.3.4.1 Parameters

CM : weight of the tractor
CJ^{fm} : distance between the front and middle axles of the tractor
CJ^{fc} : distance between the front axle and the center of gravity of the tractor
CJ^{fh} : distance between the front axle and the harness of the tractor
EM : weight of the empty trailer
EJ^{hr} : distance between the harness and the rear axle of the trailer
EJ^{cr} : distance between the center of gravity of the trailer and the rear axle
EJ^{eh} : distance between the start of the trailer and the harness
EM^{mr} : max weight on the rear axle of the trailer
EM^{mm} : max weight on the middle axle of the trailer

3.3.4.2 Variables

tm_t : weight of the stacks loaded into the truck t
ej^e : distance between center of gravity of stacks and the start of the trailer
ej^r : distance between center of gravity of stacks and the rear axle of the trailer
em^h : weight on the harness of the trailer
em^r : weight on the rear axle of the trailer
em^m : weight on the middle axle of the trailer

In the annexes, Figures 10 and 11 show the parameters and variables on the tractor and trailer.

3.3.4.3 Formula to compute the weights on harness and axles

$ej^e = \frac{\sum_{s \in \widetilde{TS}_t} (sx_s^o + \frac{(sx_s^e - sx_s^o)}{2}) \times sm_s}{tm_t}$

$ej^r = EJ^{eh} + EJ^{hr}$ - ej^e

$em^h = \frac{tm_t \times ej^r + EM \times EJ^{cr}}{EJ^{hr}}$

$em^r = tm_t + EM$ - em^h

$em^m = \frac{CM \times CJ^{fc} + em^h \times CJ^{fh}}{CJ^{fm}}$

- The weights of the stacks packed in a truck t should not exceed the truck's maximal loading weight : $\forall t \in \widetilde{T} \quad tm_t \leq TM_t^m$ with $tm_t = \sum_{s \in S_t} sm_s$ **(W1)**

- The weights on the middle axle and on the rear axle of a truck must not exceed the max weights authorized for these 2 axles : $em^m \leq EM^{mm}$ and $em^r \leq EM^{mr}$ **(W2)**

The Constraint **(W2)** must be satisfied whenever the truck is on the road. For example, if the truck pickups supplier A, then supplier B, then goes to the plant, the constraint must be satisfied (1) with only items from supplier A in the truck (drive from supplier A to B) and (2) with all the items from suppliers A and B (drive from supplier B to the plant). In other words, checking Constraint **(W2)** is done in 2 steps :

- the variables tm_{t_a}, ej^{e_a}, ej^{r_a}, em^{h_a}, em^{r_a}, em^{m_a} are calculated with only the stacks of supplier A and the Constraint **(W2$_a$)** must be satisfied. The deliveries to plant docks (inside the destination plant) are not considered as 'a road' so Constraint **(W2)** does not apply between the different plant docks.
- the variables $tm_{t_{ab}}$, $ej^{e_{ab}}$, $ej^{r_{ab}}$, $em^{h_{ab}}$, $em^{r_{ab}}$, $em^{m_{ab}}$ are calculated with the stacks of suppliers A and B, and the constraint **(W2$_{ab}$)** must be satisfied.

$$em^{m_a} \leq \text{EM}^{mm} \quad \text{and} \quad em^{r_a} \leq \text{EM}^{mr} \qquad (\mathbf{W2_a})$$

$$em^{m_{ab}} \leq \text{EM}^{mm} \quad \text{and} \quad em^{r_{ab}} \leq \text{EM}^{mr} \qquad (\mathbf{W2_{ab}})$$

4 Annexes

4.1 Placement order of suppliers and plant's dock (Constraint P4)

Figures 5 and 6 show a real-life example of placement of stacks satisfying the suppliers' pickup order and the plant docks' loading order. The first picked-up supplier 0061910800 has the plant dock G08, the second picked-up supplier 0011141500 the plant docks G08 and V09, the third picked-up supplier 0011501900 the plant docks G08 and V11 and the fourth picked-up supplier 0011579600 the plant dock G08.
Pickup order of suppliers : (1) 0061910800, (2) 0011141500, (3) 0011501900, (4) 0011579600.
Loading order of plant docks : (1) G08, (2) V11, (3) V09.

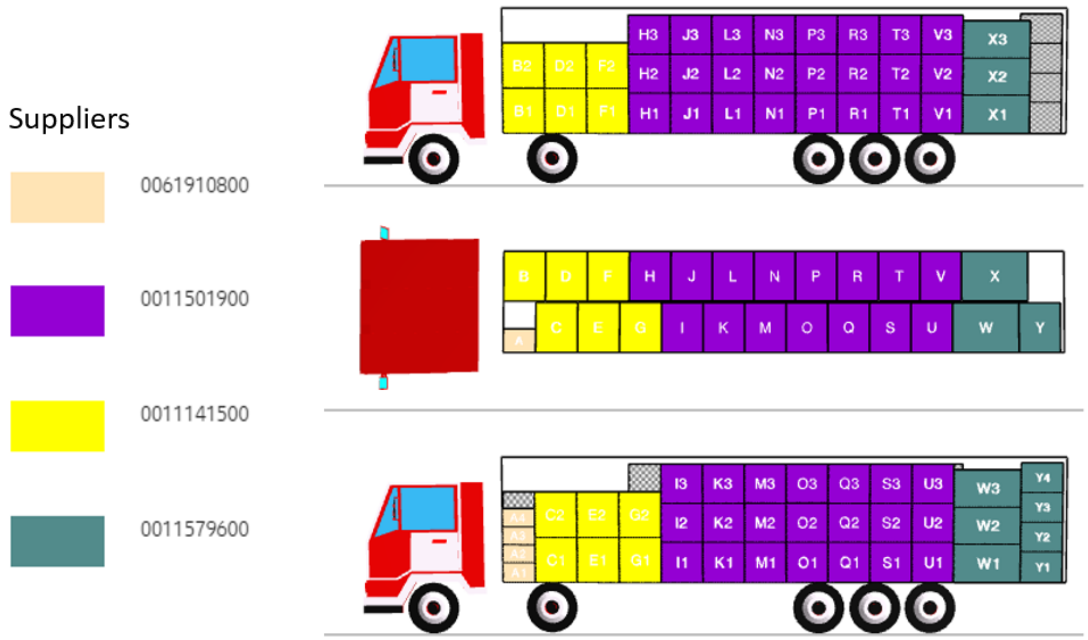

Figure 5: A truck with 4 picked-up suppliers

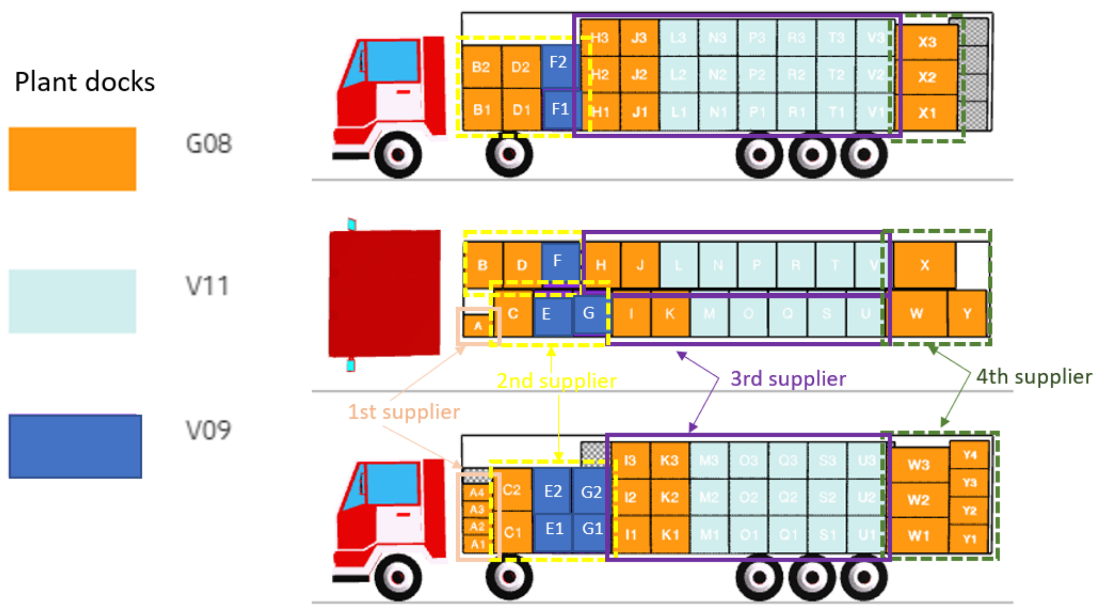

Figure 6: The same truck than Figure 5, with the plant docks for every supplier

4.2 Placement order of multi-docks stacks (Constraint P4)

Figure 7 shows a real-life example of well-placed multi-docks stacks within a single supplier. The loading order of plant docks : (1) V55, (2) G08, (3) V02, (4) V11, (5)V09, (6) V33. Stacks C, N and O have multiple plant docks. Stack C has plant docks G08 and V02, stacks O and N have plant docks V09 and V33. These couples of plant docks (G08-V02 and V09-V33) have consecutive loading orders.

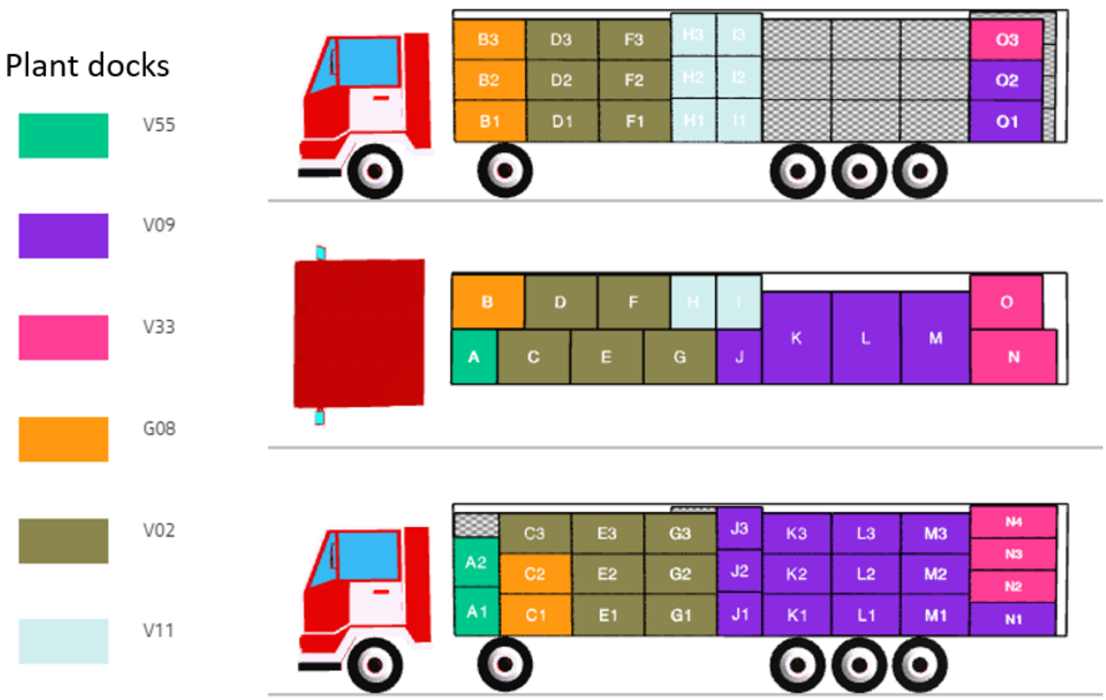

Figure 7: A truck with multi-docks stacks and a single supplier

4.3 Placement order of suppliers' docks (Constraint P4)

Figures 8 and 9 show a real-life example of placement of stacks with suppliers' docks. The truck pickups 2 suppliers, first 0090016100, then 0090015100. Supplier 0090016100 has one supplier's dock (G16), while supplier 0090015100 has 2 supplier's docks (X16868 and X16324). The loading order of supplier 0090015100 docks : (1) X16868 and (2) X16324.

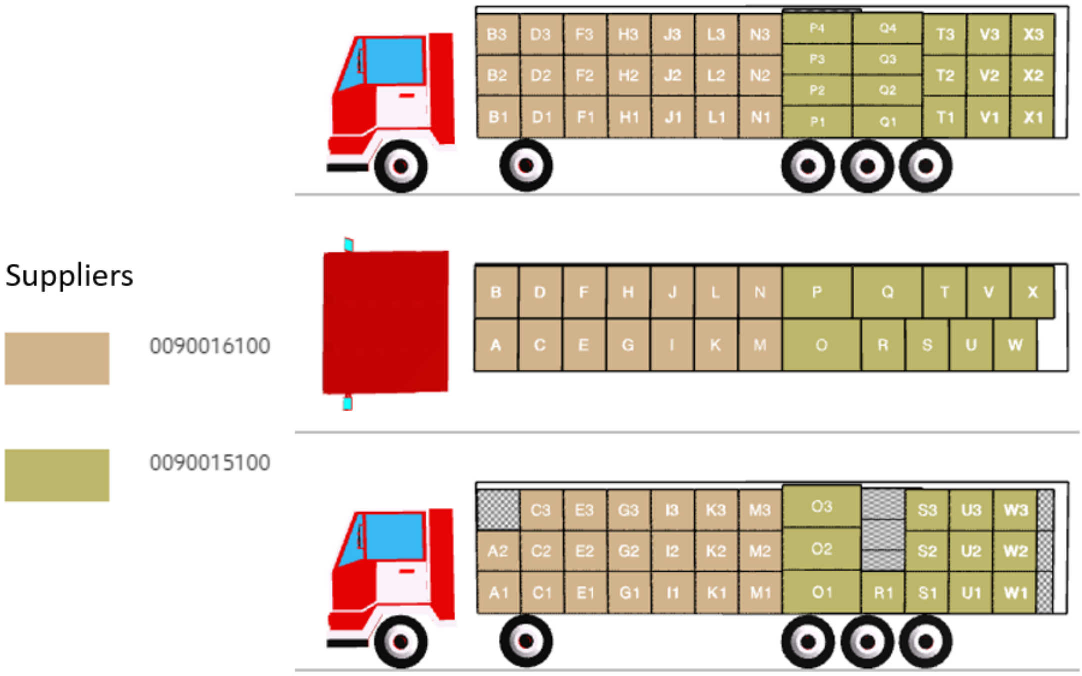

Figure 8: A truck with 2 picked-up suppliers, 0090016100 and 0090015100

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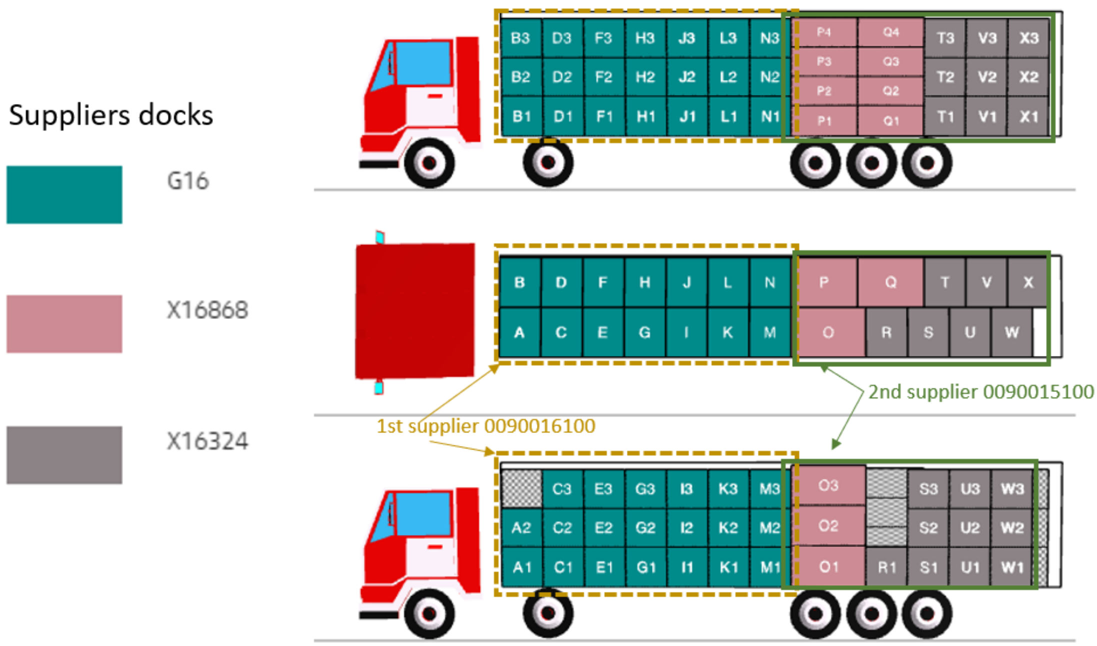

Figure 9: The same truck than Figure 8 with suppliers' docks G16, X16868 and X16324

4.4 Parameters and variables related to the tractor and the trailer of a truck

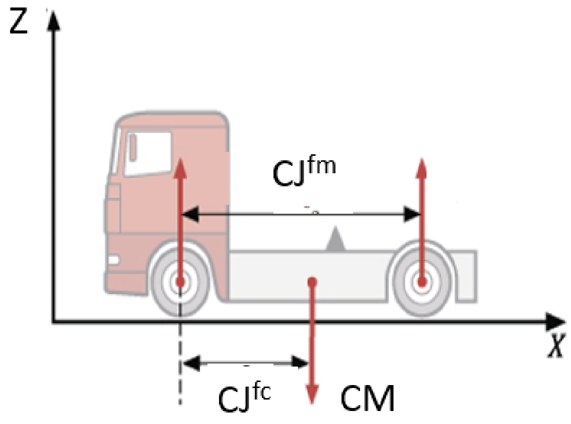

Figure 10: Parameters on the tractor

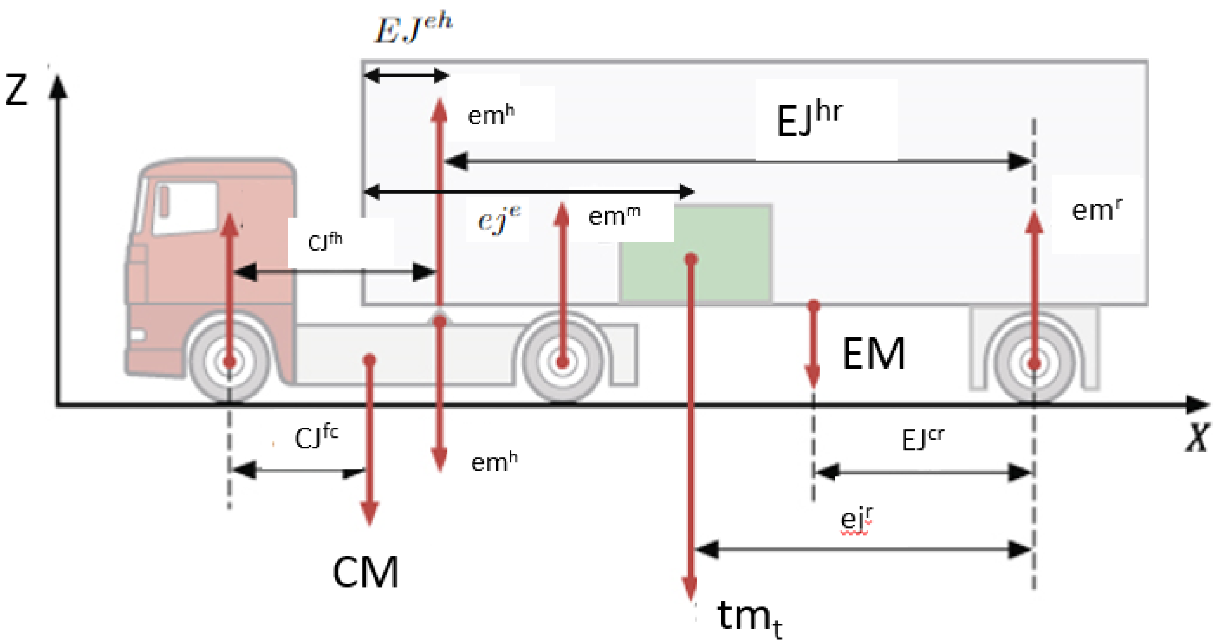

Figure 11: Parameters and variables on the trailer