EURO Gold Medal 2018
Laureate lecture

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The first international conference in operations research was held in Oxford, England, in 1957.

It was attended by 250 delegates from 21 countries

First International Conference on Operations Research – Picture of Attendees

Dedicated to all women in OR

Thanks guys!

Bernard Fortz

Patrice Marcotte

Gilles Savard

Bilevel optimization

- What is it?
- The linear case
- Bilinear objectives + linear constraints

Bilevel Optimization Problem

$$
\begin{aligned}
\max_{x,y} \quad & f(x,y) \\
\text{s.t.} \quad & (x,y) \in X \\
& y \in S(x) \\
\text{where} \quad & S(x) = \operatorname*{argmax}_{y} g(x,y) \\
& \text{s.t.} (x,y) \in Y
\end{aligned}
$$

First paper on bilevel optimization

Bracken & McGill (OR, 1973): First bilevel model, structural properties, military application.

Mathematical Programs with Optimization Problems in the Constraints

Jerome Bracken and James T. McGill

Institute for Defense Analyses, Arlington, Virginia

(Received October 5, 1971)

This paper considers a class of optimization problems characterized by constraints that themselves contain optimization problems. The problems in the constraints can be linear programs, nonlinear programs, or two-sided optimization problems, including certain types of games. The paper presents theory dealing primarily with properties of the relevant functions that result in convex programming problems, and discusses interpretations of this theory. It gives an application with linear programs in the constraints, and discusses computational methods for solving the problems.

Adequate framework for Stackelberg game

- Leader: 1st level,
- Follower: 2nd level,
- Leader takes follower’s optimal reaction into account.

Heinrich von Stackelberg
(1905 - 1946)

Applications

- Economic game theory
- Production planning
- Revenue management
- Security
- …

Linear BP

$$\max_{x,y} \quad c_1 x + d_1 y$$
$$\text{s.t.} \quad A_1 x + AB_1 y \leq b_1$$
$$\max_y d_2 y,$$
$$\text{s.t.} A_2 x + B_2 y \leq b_2$$

Linear BP

$$
\begin{aligned}
\max_{x} \quad & c_1 x + d_1 y \\
\text{s.t.} \quad & A_1 x + B_1 y \leq b_1 \\
& B_2 y + A_2 x \leq b_2 \\
& \lambda B_2 = d_2 \\
& \lambda \geq 0 \\
& \boxed{\lambda (B_2 y - b_2 + A_2 x) = 0}
\end{aligned}
$$

$$
\begin{aligned}
\max_{x} \quad & c_1 x + d_1 y \\
\text{s.t.} \quad & A_1 x + B_1 y \leq b_1 \\
& B_2 y + A_2 x \leq b_2 \\
& \lambda B_2 = d_2 \\
& \lambda \geq 0 \\
& \lambda \leq M_d z \\
& A_2 x + B_2 y \geq b_2 - M_p (1 - z) \\
& z \in \{0, 1\}^m
\end{aligned}
$$

Fortuny-Amat, McCarl (1981)

Linear BP:what is true

- If Linear BP is feasible, then there exists an optimal solution which is a **vertex** of the polyhedron defined by both levels constraints.

- There exists a finite value for **M**

Linear BP: what is the problem with **M**?

- "Trial and error" does not work to determine **M** (Pineda, Morales, 2019)

- Finding valid **M** is NP-hard (Kleinert, Labbé, Plein, Schmidt, 2019)

Price Setting Problem with linear constraints

$$\max_{x, y_1, y_2} \quad xy_1$$

$$\min_{y_1, y_2} \quad (c + x)y_1 + dy_2$$

$$\text{s.t.} \quad Ay_1 + By_2 = b$$

$$y_1, y_2 \geq 0$$

Price Setting Problem

$$\max_{y_1, y_2, \lambda} \quad \lambda b \quad - \quad (c_1 y_1 + c_2 y_2)$$

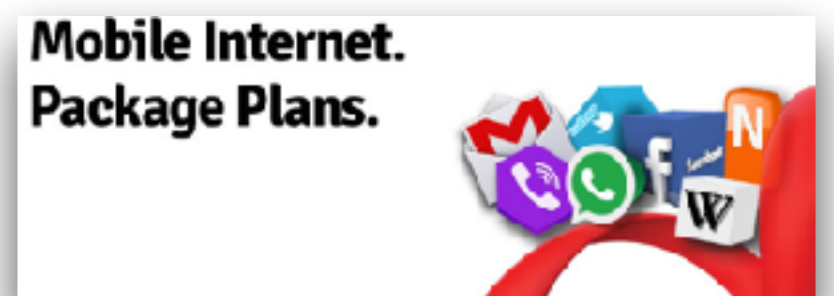
$$\text{s.t.} \quad \lambda A_2 \leq c_2 \qquad \text{s.t.} \quad A_1 y_1 + A_2 y_2 = b$$

$$y_1, y_2 \geq 0$$

$$(c_2 - \lambda A_2) y_2 = 0$$

Labbé, Marcotte, Savard (2000)

Applications



Network pricing problem

(Labbé, Marcotte, Savard, 1998)

- network with toll arcs (A_1) and non toll arcs (A_2)
- Costs c_a on arcs
- Commodities (o^k, d^k, n^k)
- Routing on cheapest (cost + toll) path
- Maximize total revenue

Solution approach

- MILP formulation
- Tight **M** very effective
- Branch & cut

Conclusion

- Bilevel optimization has a bright future!
- Be aware : lots of fake news!
- Need of generic softwares (handling properly complementarity)
- Need to exploit problem inner structure !!


Thank You
Arigato
Grazie
Merci
Spasiba
Cheers
Efharisto
Danke
Gracias
Hvala
Toda
Matondo
Gratzias
Köszönöm
Takk
Nuhun
Misaotra
Madlobt
Tenki
Mahalo
Aitäh
Saha
Efharisto
Barkal
Blagodaria
Mauruuru
Soolong
Sobodi
Dankewol
Skee
Kiitos
Paldies
Murakoze
Trugéré
Marahaba
Dekoju
Shukria
Dekuji
Multumesc
Bedankt
Obrigada
Meharbani
Shukran
Blagodaram
Tanmirt
Menlau
Aabar
Tanan
Grassie
Danki
Salamat
Obrigado
Thank You
Waita
Nandri
Matondi
Modupe
Omol
Spas
Akiba
Dziekuje
Manjuthe
Dakujem
Stuutiyi
Vinaka
Rahmat
Chokrane
Shakkran
Mauruuru
Tanemirt
Manana
Murakoze
Merkzi
Gràcies
Tashakor
Tack
Mèsi
Nkosi
Buznyg
Zikomo
Kinisou
Tanmirt
Bayarlalaa
Ahsante
Dziakuju
Yekeniele
Miigwech
Dankegon
Ngiyabonga
Manana
Shukriyaa
Talofa
Welalin
Sulpáy
Multumesc