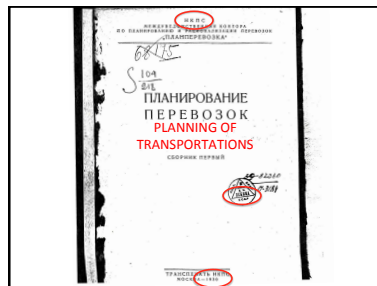




Operational Research ↔ Transportation

→ Tolstol 1930: min-cost transportation
 Kantorovich 1939: "Problem C"
 Hitchcock 1941: transportation problem
 Koopmans 1942: transshipment problem
 Dantzig 1947: linear programming



Метод нахождения наименьшего суммарного километража при планировании перевозок в пространстве.

Допустим, имеем несколько городов, расположенных на плоскости. Известны расстояния между всеми городами. Требуется найти маршрут, соединяющий все города, с наименьшим суммарным километражем. Это задача нахождения минимального остовного дерева графа, вершины которого являются городами, а ребра — расстояния между ними.

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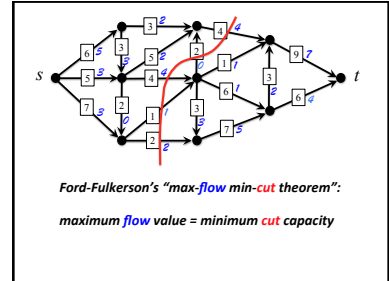
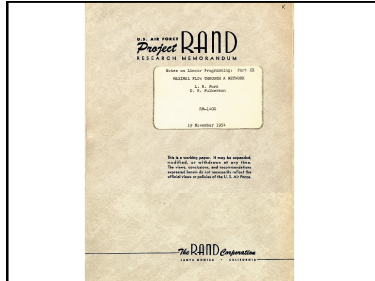
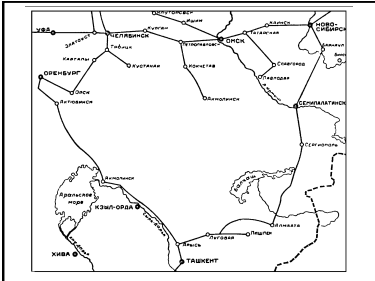
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| № | Наименование | Единица измерения | Количество | ... |
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MAXIMAL FLOW THROUGH A NETWORK

L. R. Ford
D. R. Fulkerson

Introduction

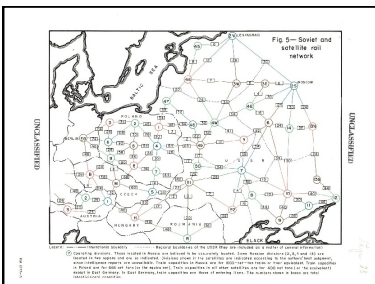
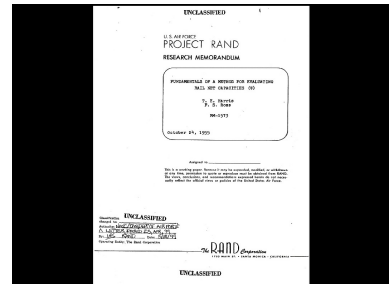
The problem discussed in this paper was formulated by T. Harris as follows:

"Consider a rail network connecting two cities by way of a number of intermediate cities, where each link of the network has a number assigned to it representing its capacity. Assuming a steady state condition, find a maximal flow from one given city to the other."

While this can be set up as a linear programming problem with as many equations as there are cities in the network, and hence can be solved by the simplex method, [1], it turns out that in the cases of most practical interest, where the network is planar in a certain restricted sense, a much simpler and more efficient hand computing procedure can be described.

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